Tesis doctoral

DESARROLLO DE UN MODELO DE FORMULACIÓN SEMIANALÍTICA PARA SU APLICACIÓN EN ARRAYS BIDIMENSIONALES DE OSCILADORES ACOPLADOS

Autor

Pedro Umpiérrez Rodríguez

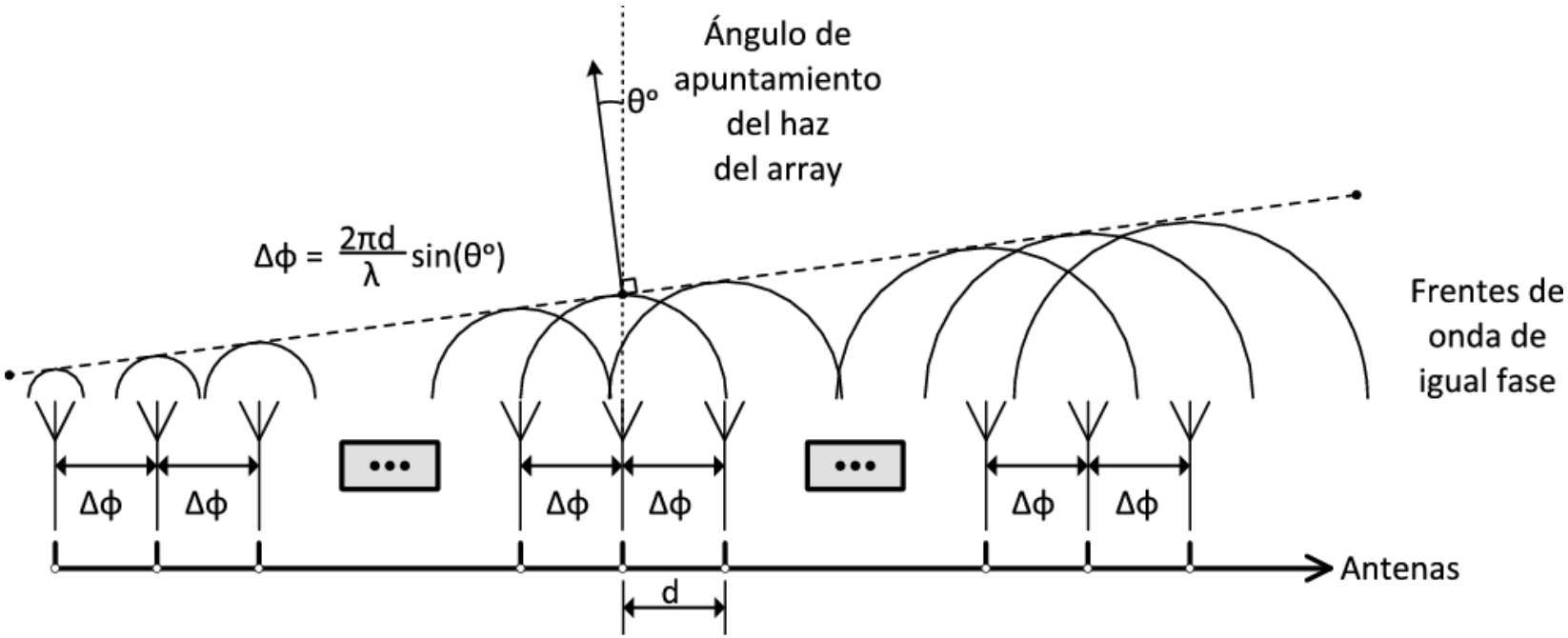
Tutores

Víctor A. Araña Pulido

Sergio Sancho Lucio

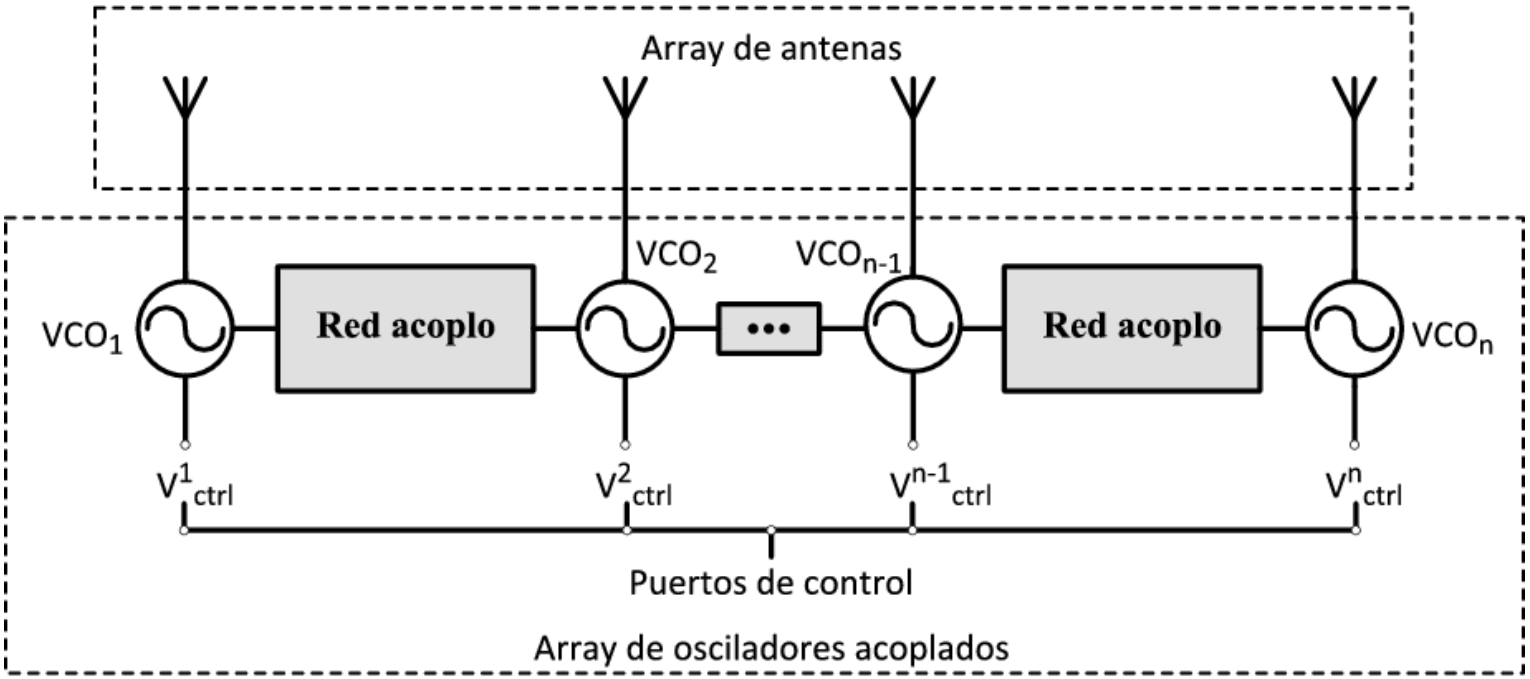
Motivación

- Array de antenas



Motivación

- Array de osciladores acoplados



Motivación

- Formulación semianalítica (FSA)
  - Modelo de orden reducido
  - Aproximación funcional
  - Disminuye el número de parámetros

- Aplicación al análisis de arrays de osciladores lineales

Objetivo

Extender el uso de la FSA a los arrays bidimensionales de osciladores acoplados:

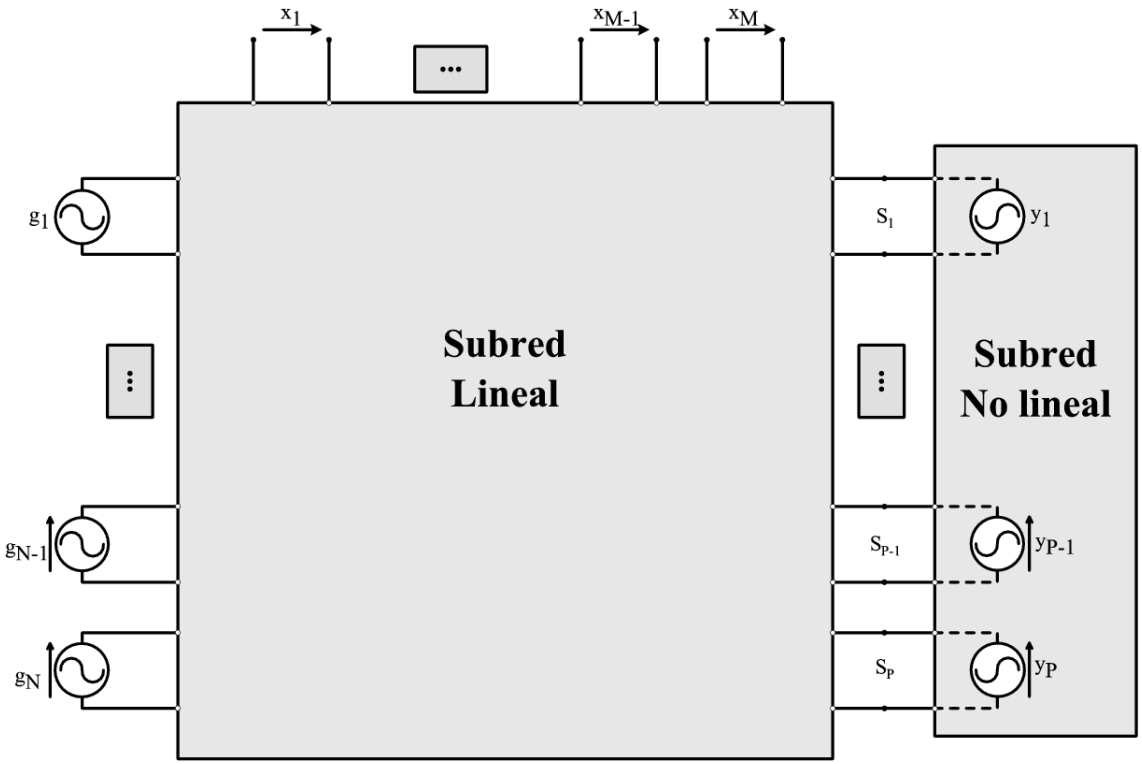
- Formulación general
- Modos de funcionamiento, estabilidad y ruido
- Modificaciones de la FSA para casos específicos
- Cálculo experimental de las derivadas del oscilador

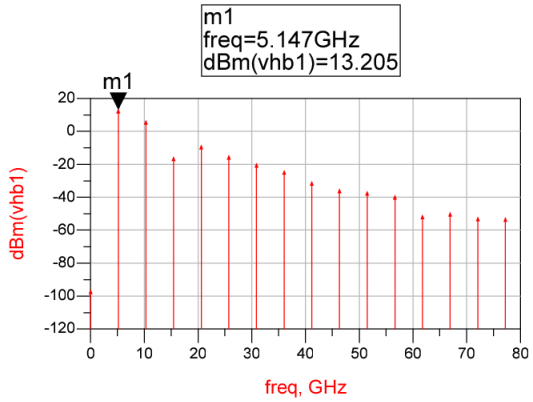
Índice

- Conceptos previos
- Análisis de arrays
- Redes de acoplo
- Ajustes aplicables a la FSA
- Aplicación práctica
- Medida de las derivadas

Conceptos previos

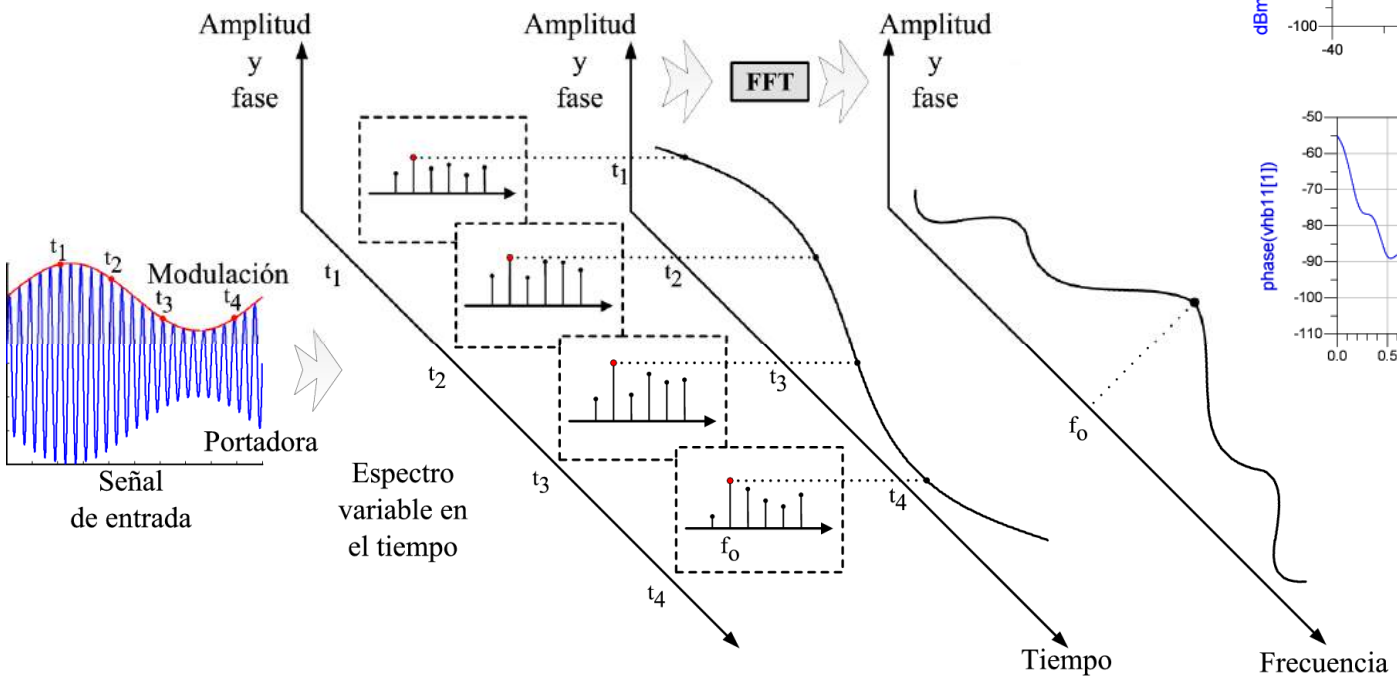
- Balance armónico

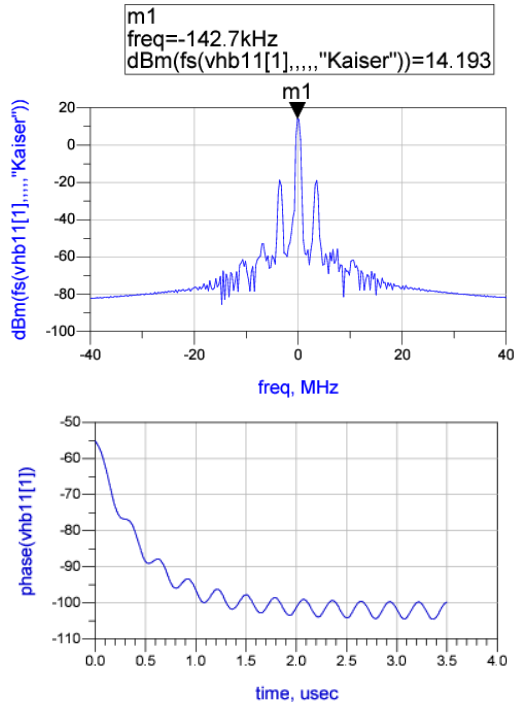




Conceptos previos

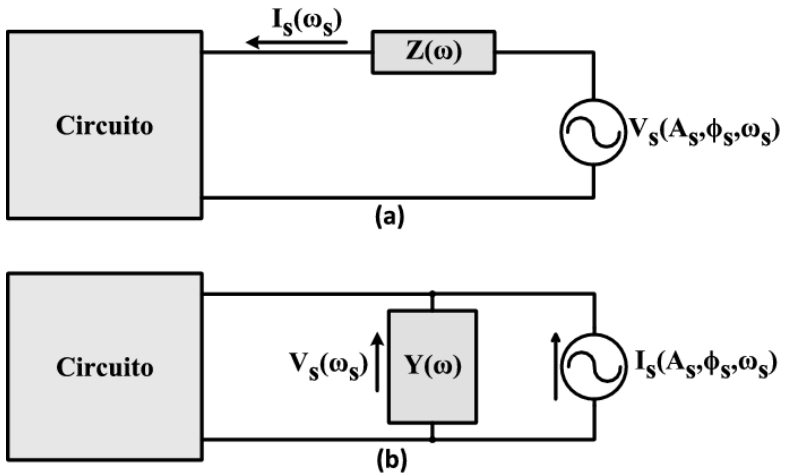
- Envolvente compleja





Conceptos previos

- Generador auxiliar



- Cálculo de derivadas

$$\frac{\delta Y_{out}}{\delta V_s} \approx \frac{Y_{out}\left(V_s + \triangle V_s,\, \phi_s,\, \omega_s\right) - Y_{out}\left(V_s - \triangle V_s,\, \phi_s,\, \omega_s\right)}{2\triangle V_s}$$

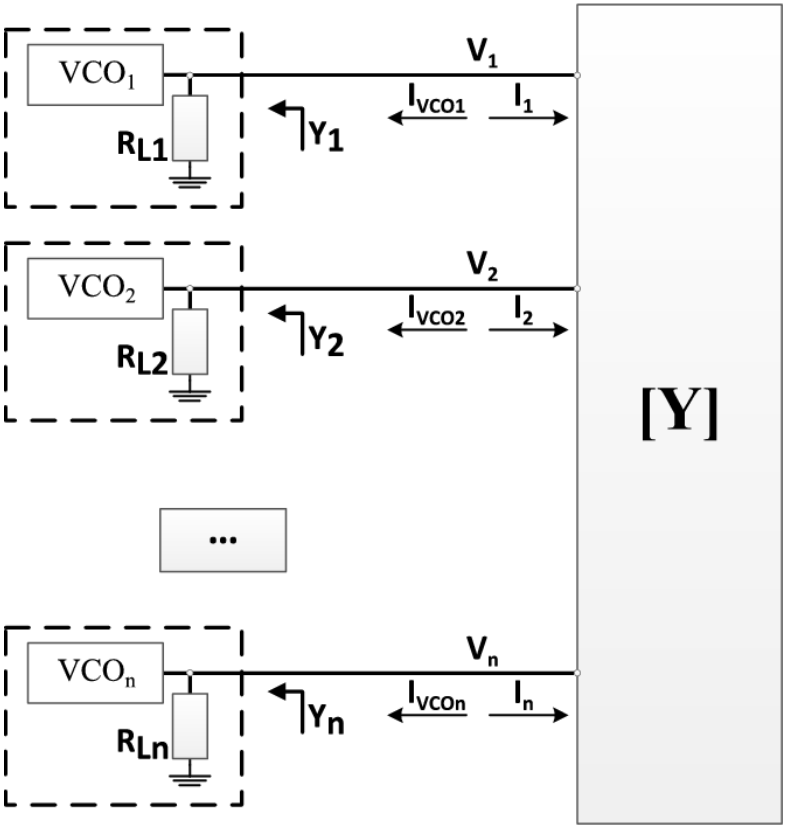
[A. Suarez et all, 2011]

Análisis de arrays con la FSA

- Obtención de las soluciones estacionarias
  - Régimen libre

$$V_i Y_i + \sum_{k=1}^{n} V_k y_{ik} = 0$$

$$(i = 1 \cdots n)$$



Análisis de arrays con la FSA

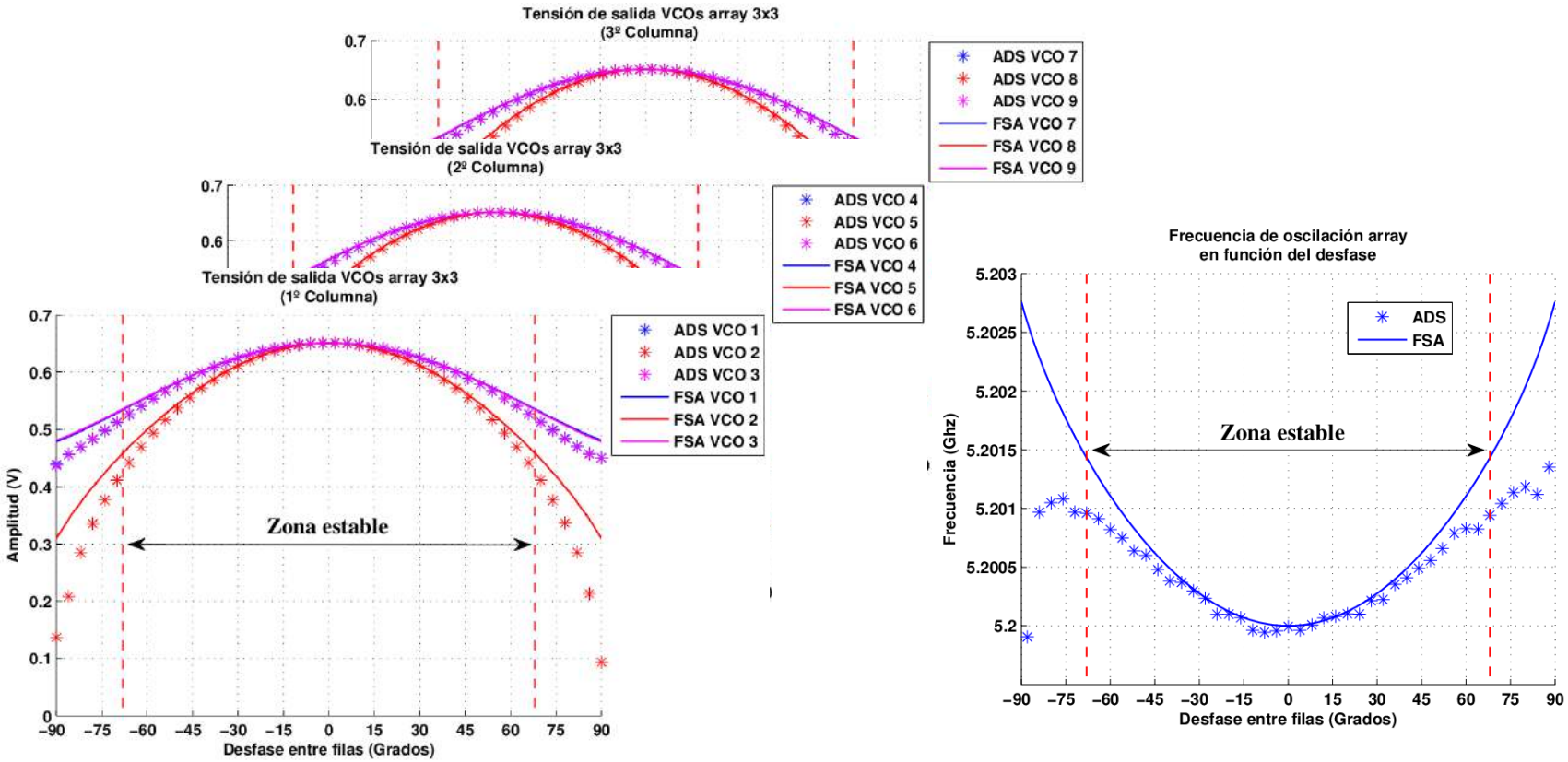
- Aproximación Taylor de la admitancia de los VCOs

$$\left(\frac{\delta Y}{\delta V}\left(V_i - V_o\right) + \frac{\delta Y}{\delta \omega}\left(\omega - \omega_o\right) + \frac{\delta Y}{\delta \eta}\left(\eta_i - \eta_o\right)\right) V_i e^{j\phi_i} + \\ + \sum_{k=1}^{n} y_{ik}\left(\omega\right) V_k e^{j\phi_k} = 0$$

$$\left(i = 1 \cdots n\right)$$

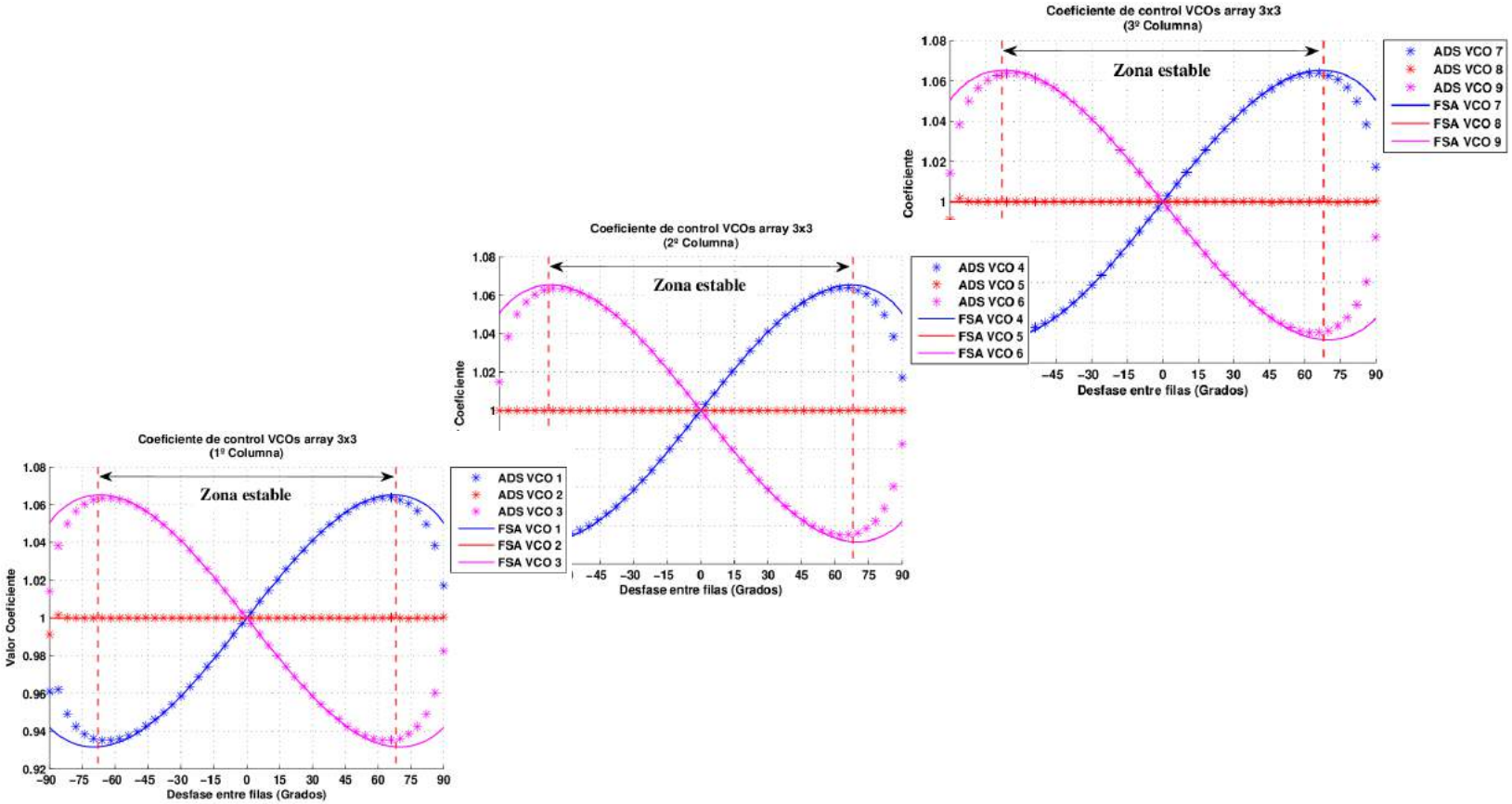
Análisis de arrays con la FSA

- Soluciones: amplitud y frecuencia



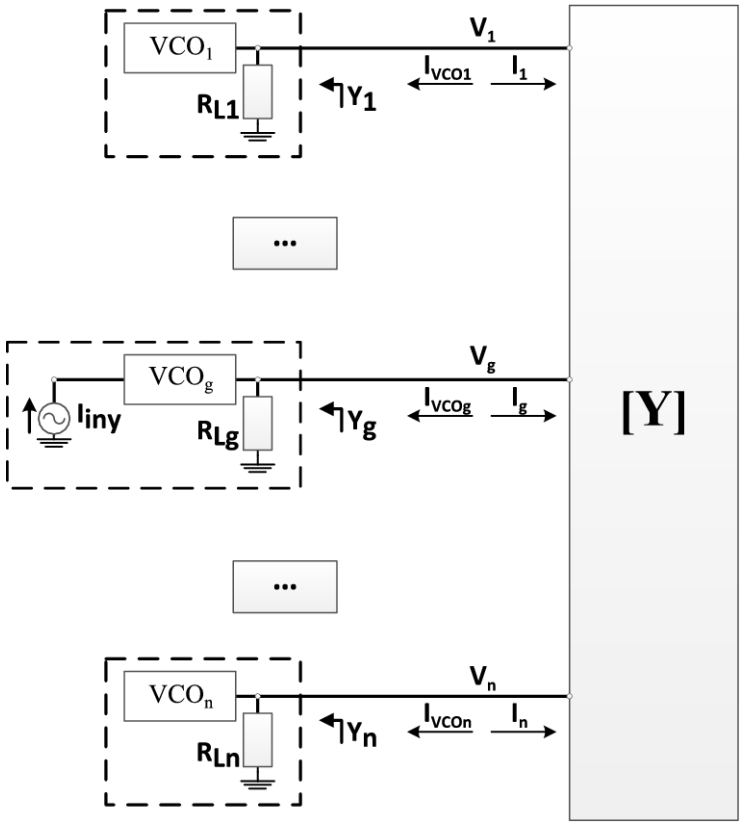
Análisis de arrays con la FSA

- Soluciones: parámetro de control



Análisis de arrays con la FSA

- Régimen inyectado



$$\left\{\frac{\delta Y}{\delta V}\left(V_i - V_o\right) + \frac{\delta Y}{\delta \omega}\left(\omega_{iny} - \omega_o\right) + \frac{\delta Y}{\delta \eta}\left(\eta_i - \eta_o\right) + \right.$$

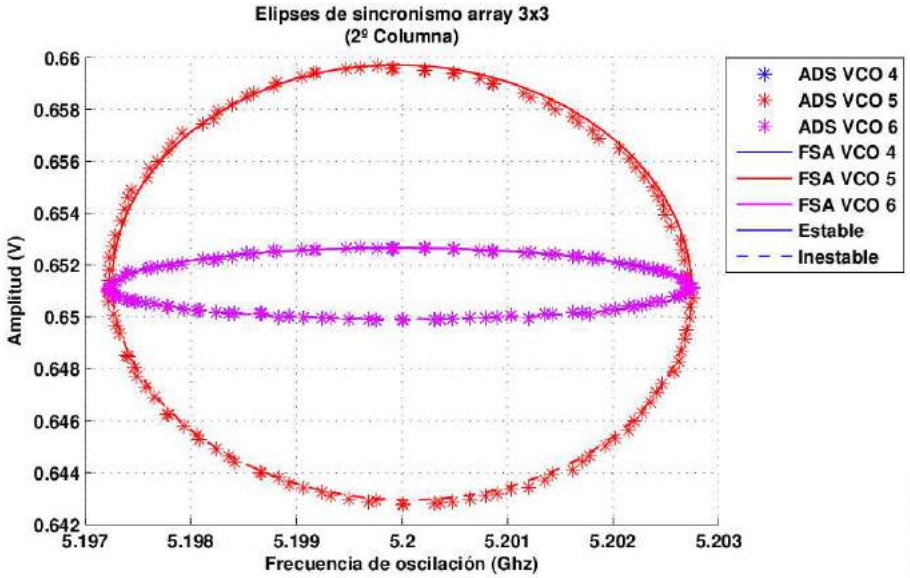
$$\left. + I_{iny}\delta_g^i\left(\frac{\delta Y}{\delta I^r}\cos\left(\phi_{iny} - \phi_i\right) + \frac{\delta Y}{\delta I^i}\sin\left(\phi_{iny} - \phi_i\right)\right)\right\} V_i e^{j\phi_i} +$$

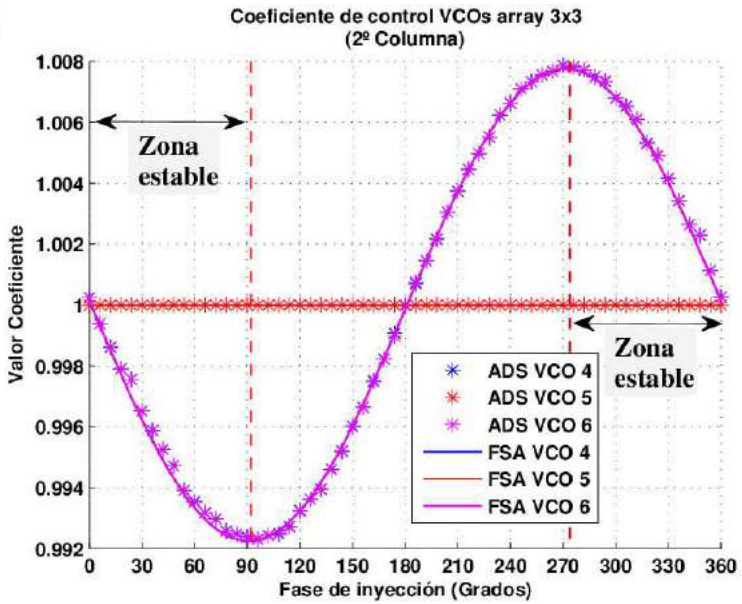
$$+ \sum_{k=1}^{n} y_{ik}\left(\omega_{iny}\right) V_k e^{j\phi_k} = 0$$

$$\left(i = 1 \cdots n\right)$$

Análisis de arrays con la FSA

- Soluciones régimen inyectado





Análisis de arrays con la FSA

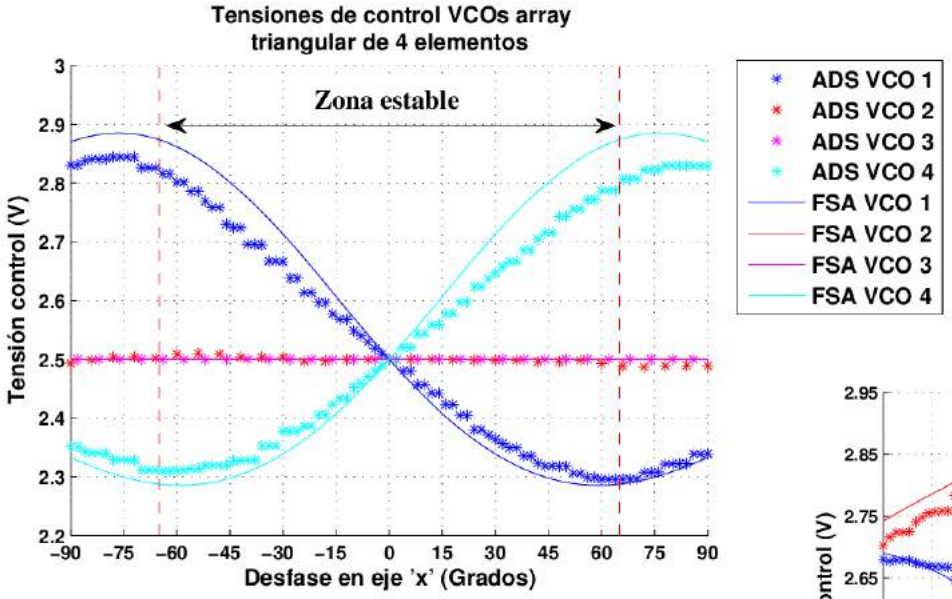
- Ajuste del sistema
  - Se fijan las fases de las tensiones en función de la topología del array $\phi_i = f_i\left(\bar{\Theta}\right)$
  - El resto de grados de libertad se emplean en los parámetros de control

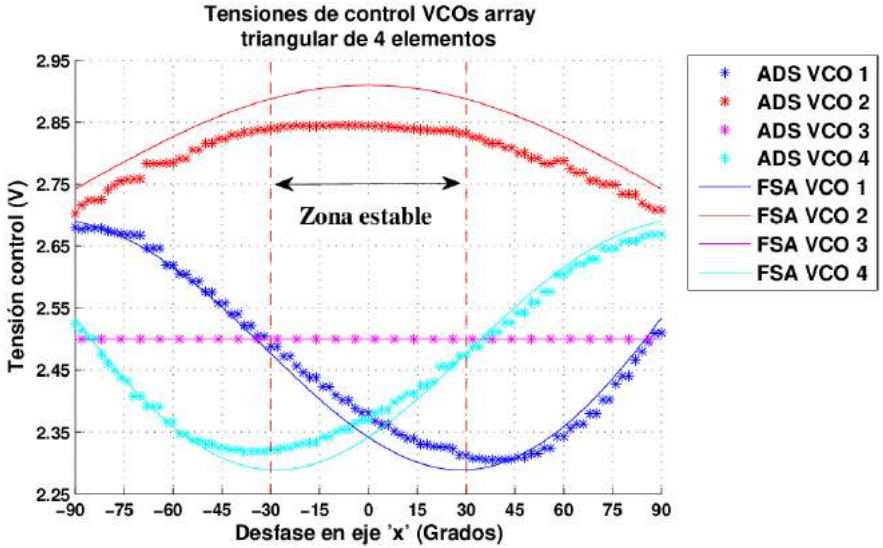
Análisis de arrays con la FSA

- Análisis de estabilidad
  - Se obtiene a partir de las soluciones estacionarias
  - Evalúa la evolución de las soluciones a lo largo del tiempo frente a perturbaciones aleatorias de pequeña amplitud en cada una de las variables del sistema
  - Si la solución es estable el sistema converge de nuevo a la solución estacionaria tras la perturbación.

Análisis de arrays con la FSA

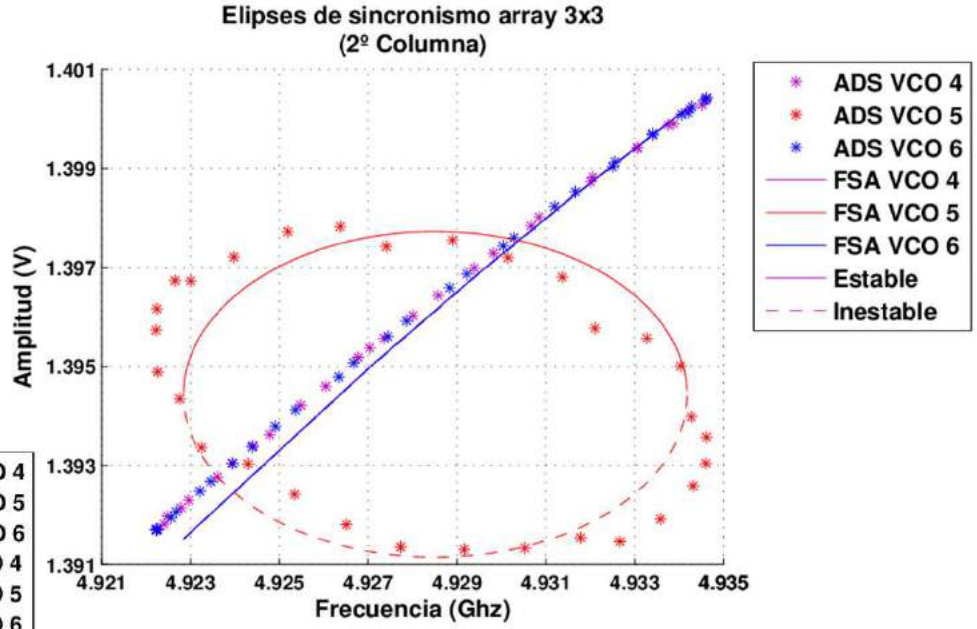
- Régimen libre

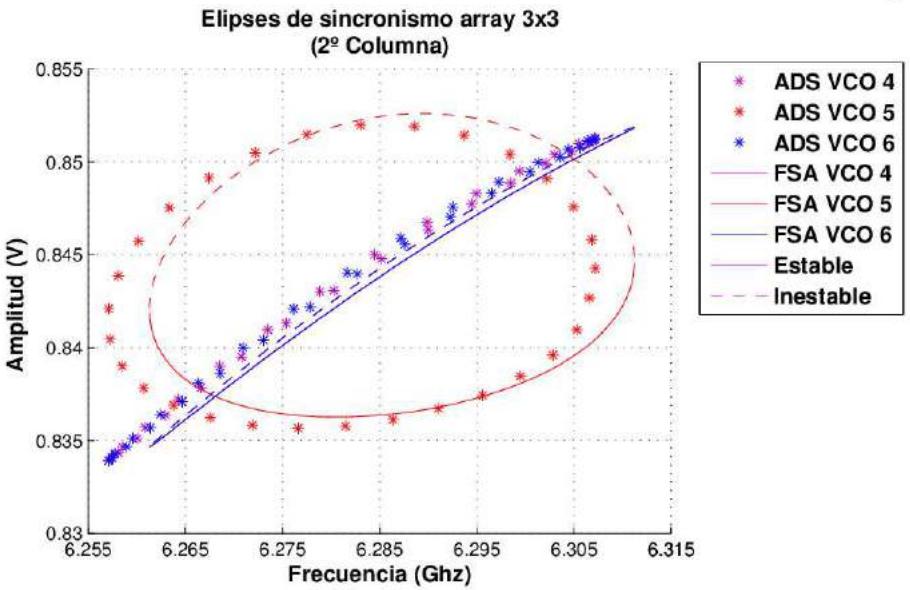




Análisis de arrays con la FSA

- Régimen inyectado



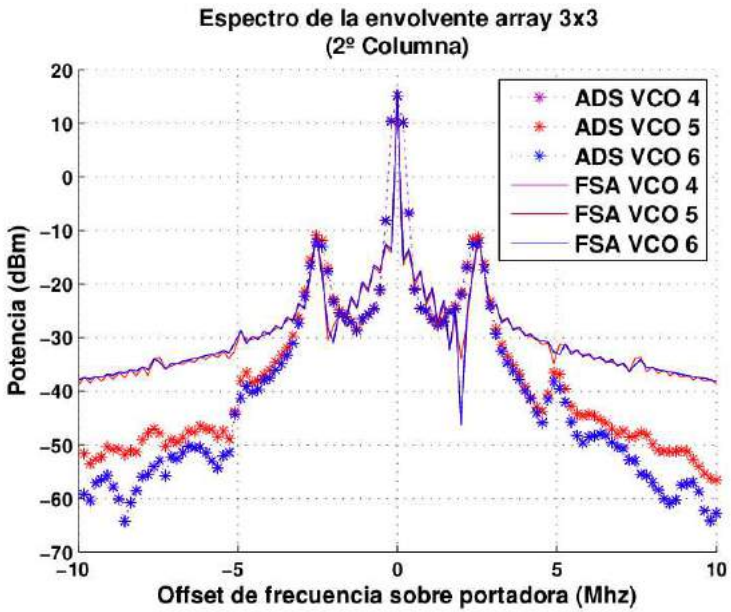


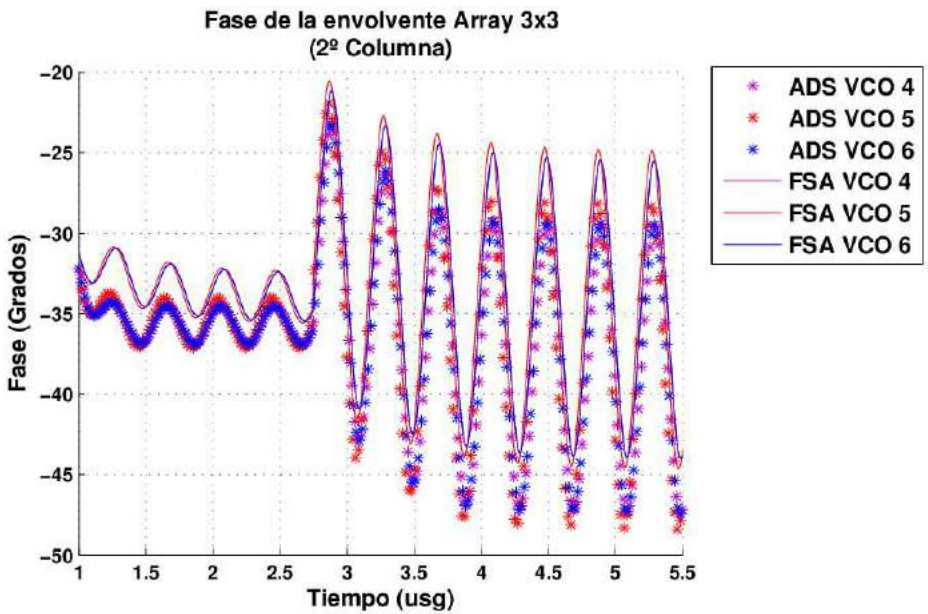
Análisis de arrays con la FSA

- Envolvente compleja
  - Analiza la evolución de las soluciones en el tiempo cuando se inyecta el sistema con señales moduladas $\phi_{iny}(t) = f_{mod}(t)$
  - Resuelve el sistema mediante un método de integración directa en el tiempo.

Análisis de arrays con la FSA

- Soluciones



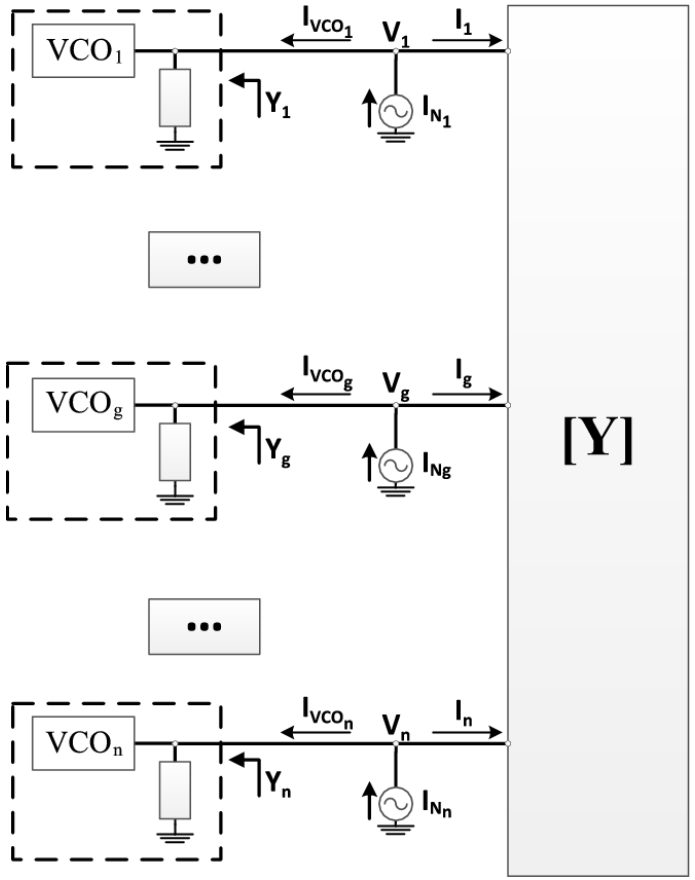


Análisis de arrays con la FSA

- Ruido de fase
  - Se obtiene a partir de una caracterización previa de los osciladores aislados
  - Permite determinar como afecta el sincronismo a los niveles de ruido en el sistema

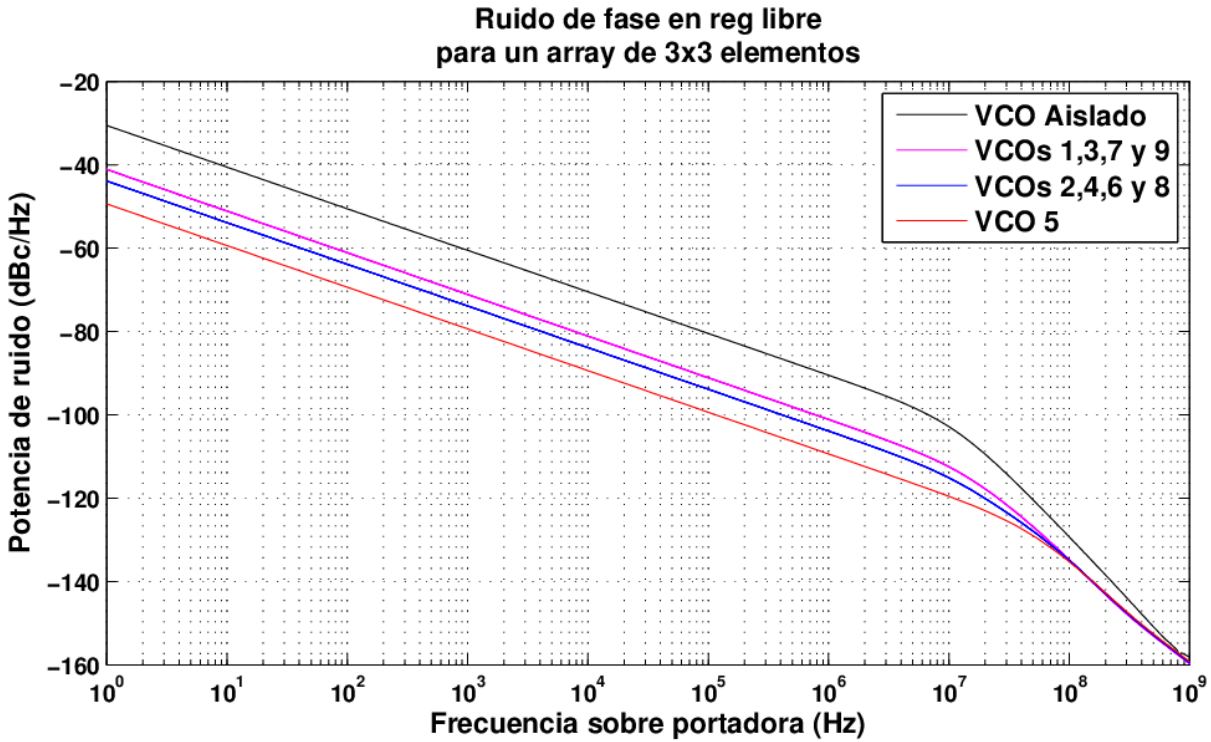
Análisis de arrays con la FSA

- Régimen libre
  - Introduce una fuente de ruido equivalente en cada uno de los VCOs
  - Se emplean dos componentes armónicas: fundamental y DC



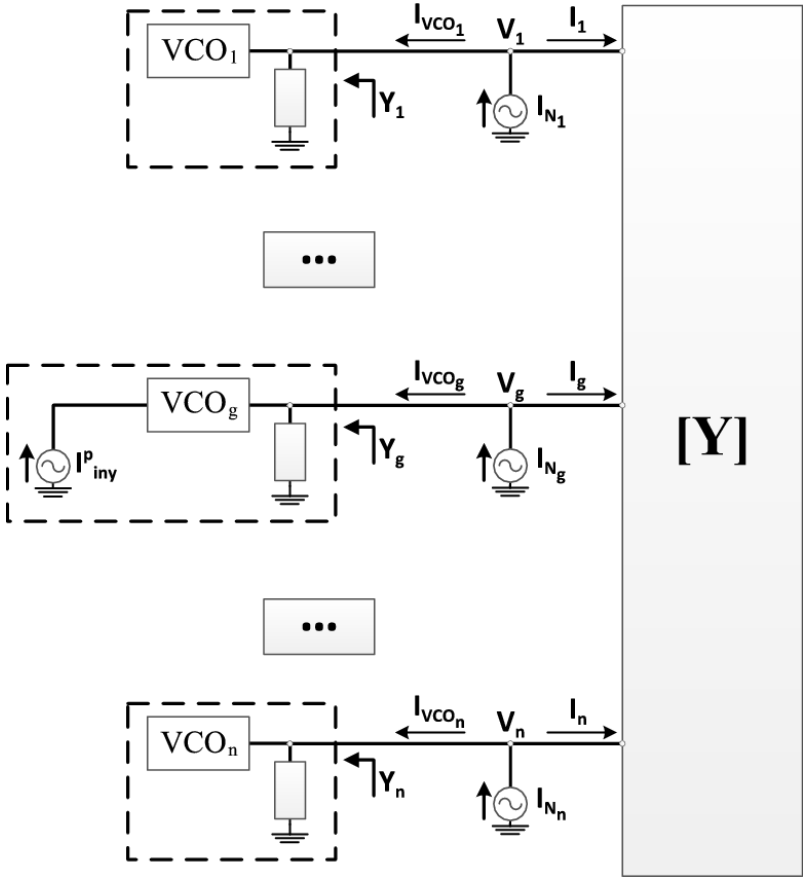
Análisis de arrays con la FSA

- Soluciones



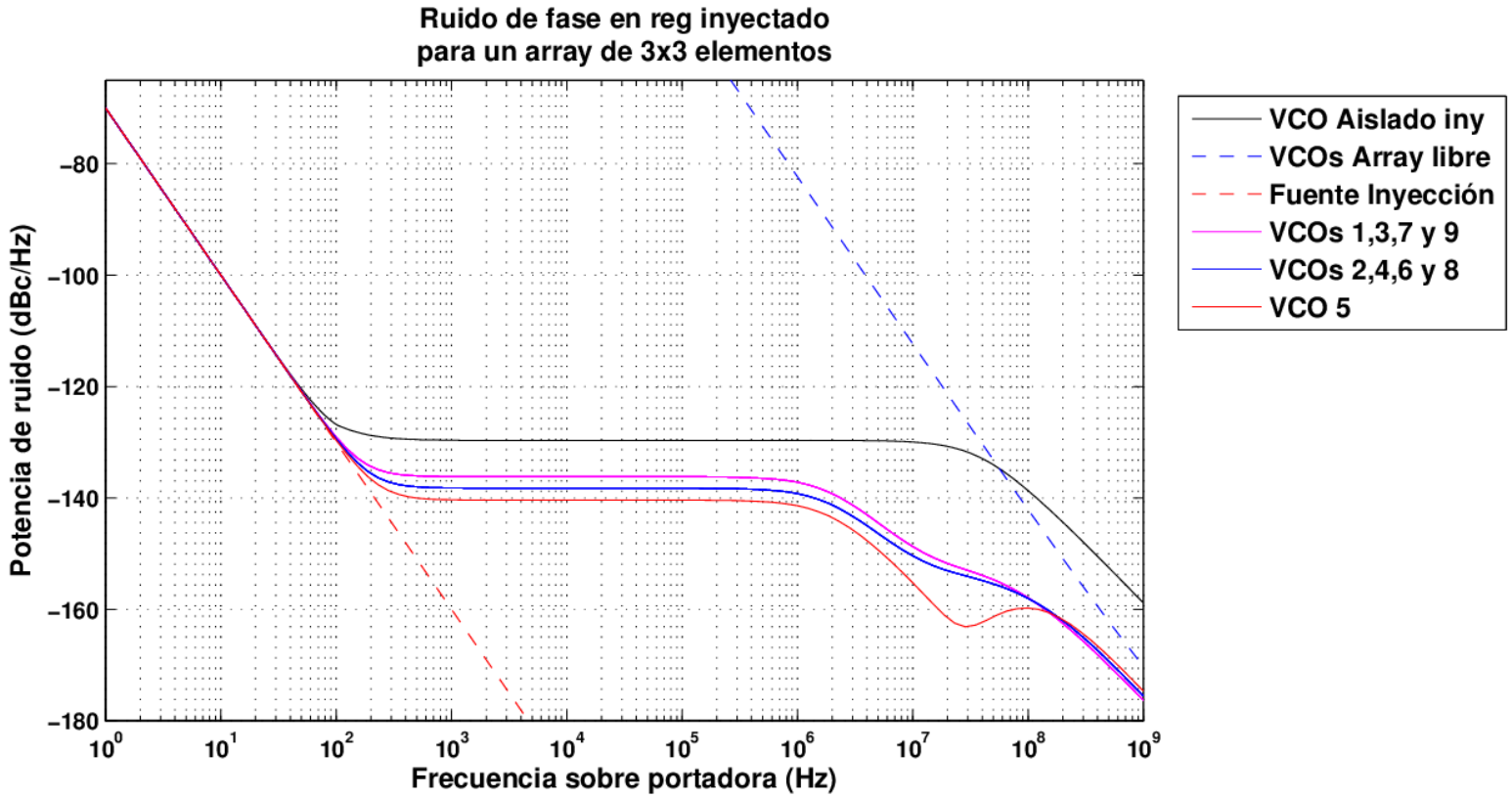
Análisis de arrays con la FSA

- Régimen inyectado
  - Contempla el ruido blanco de los VCOs y el ruido de fase propio de la señal de inyección



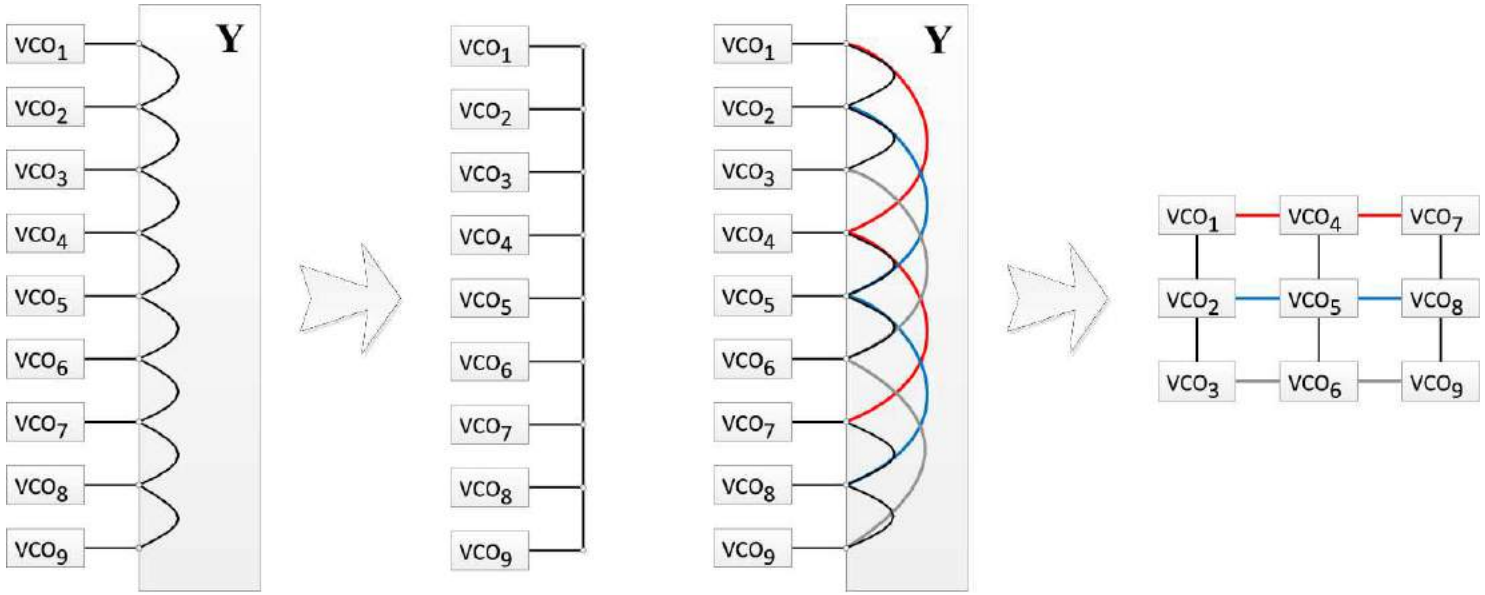
Análisis de arrays con la FSA

- Soluciones



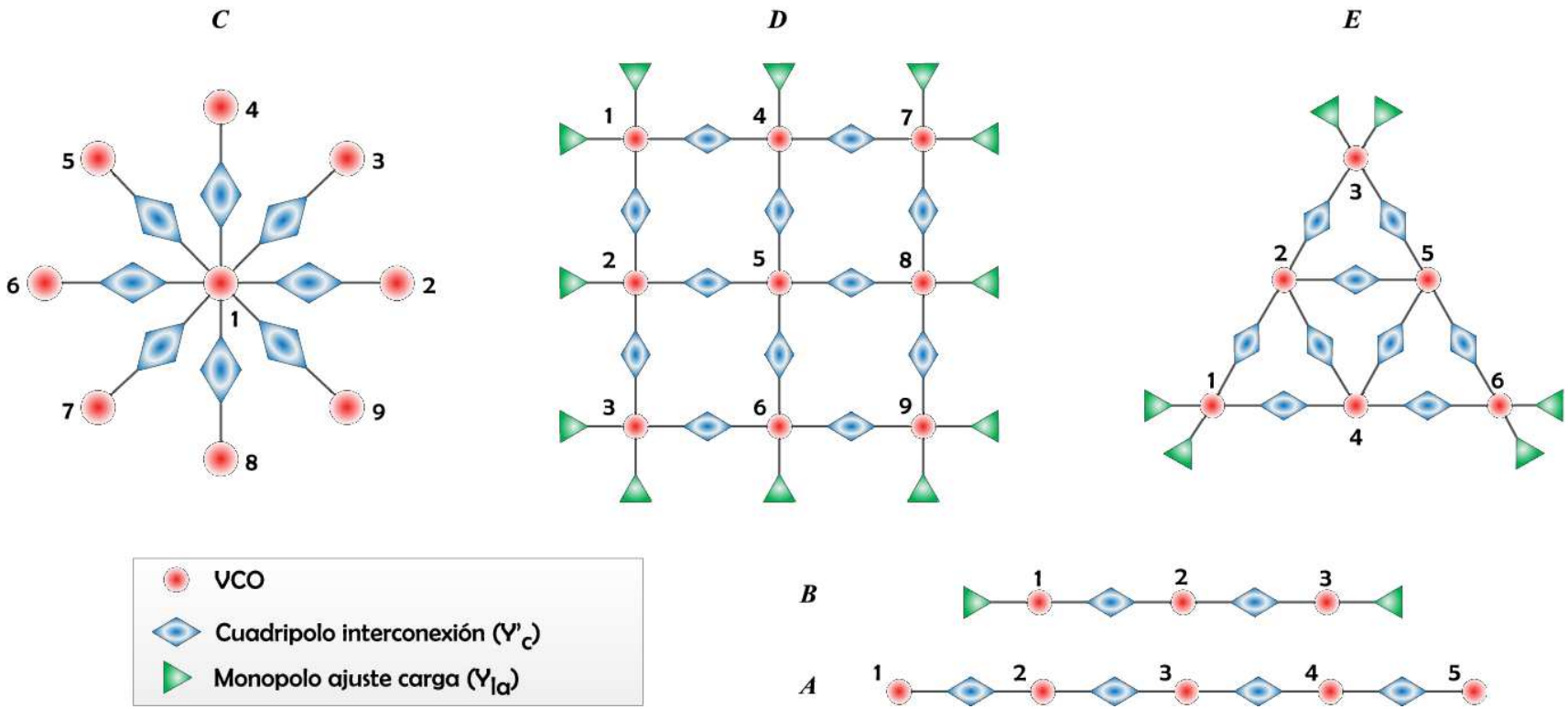
Redes de acoplo

- Interconecta a los VCOs entre sí
- Determina la topología del array



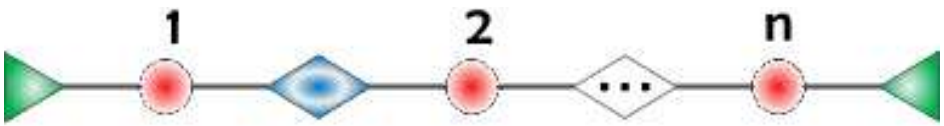
Redes de acoplo

- Formada por la unión de cuadripolos



Redes de acoplo

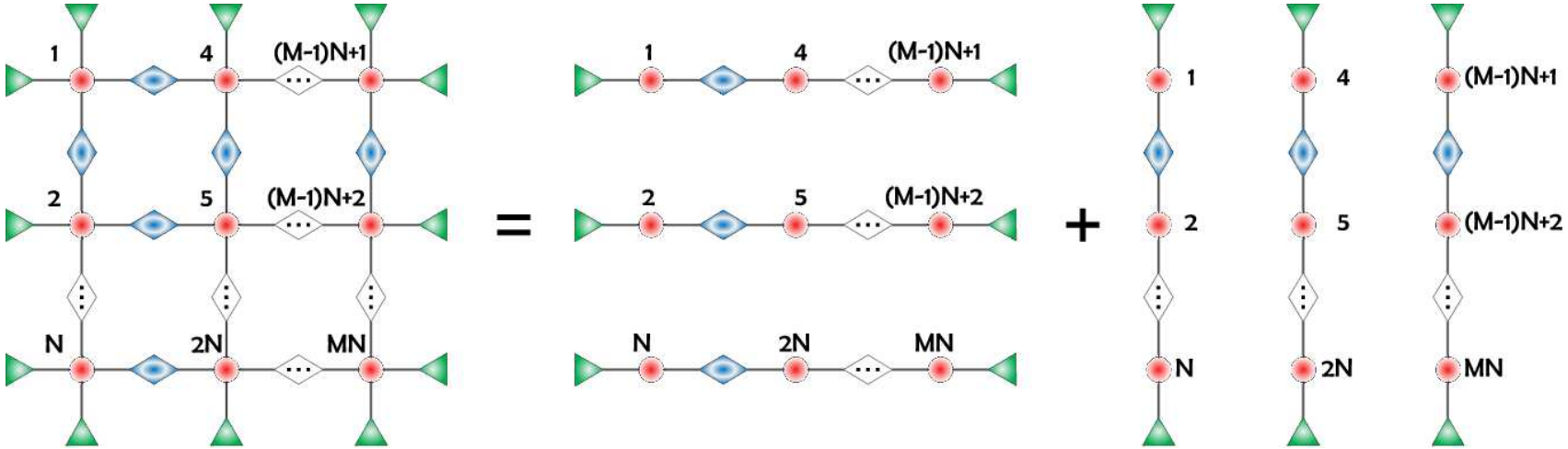
- Array lineal



$$\bar{\bar{Y}}_{gl}\Big|_{(nxn)} = \begin{bmatrix} 2y_r & y_t & 0 & 0 & \cdots & 0 & 0 \\ y_t & 2y_r & y_t & 0 & \cdots & 0 & 0 \\ 0 & y_t & 2y_r & y_t & \cdots & 0 & 0 \\ 0 & 0 & y_t & 2y_r & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & y_t & 0 \\ 0 & 0 & 0 & 0 & \cdots & 2y_r & y_t \\ 0 & 0 & 0 & 0 & \cdots & y_t & 2y_r \end{bmatrix}$$

Redes de acoplo

- Array bidimensional rectangular



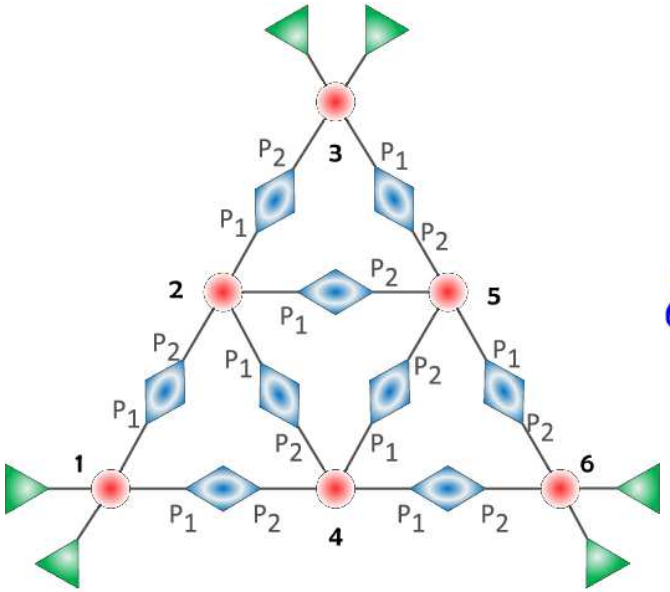
Redes de acoplo

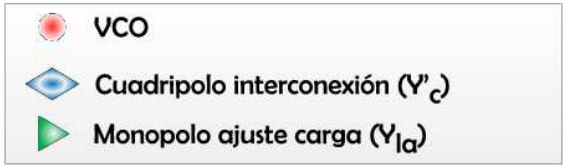
- Ej: Array rectangular $3x3$

$$\bar{\bar{Y}}_{gl}\Big|_{(9x9)} = \begin{bmatrix} 4y_r & y_t & 0 & y_t & 0 & 0 & 0 & 0 & 0 \\ y_t & 4y_r & y_t & 0 & y_t & 0 & 0 & 0 & 0 \\ 0 & y_t & 4y_r & 0 & 0 & y_t & 0 & 0 & 0 \\ y_t & 0 & 0 & 4y_r & y_t & 0 & y_t & 0 & 0 \\ 0 & y_t & 0 & y_t & 4y_r & y_t & 0 & y_t & 0 \\ 0 & 0 & y_t & 0 & y_t & 4y_r & 0 & 0 & y_t \\ 0 & 0 & 0 & y_t & 0 & 0 & 4y_r & y_t & 0 \\ 0 & 0 & 0 & 0 & y_t & 0 & y_t & 4y_r & y_t \\ 0 & 0 & 0 & 0 & 0 & y_t & 0 & y_t & 4y_r \end{bmatrix}$$

Redes de acoplo

- Red genérica

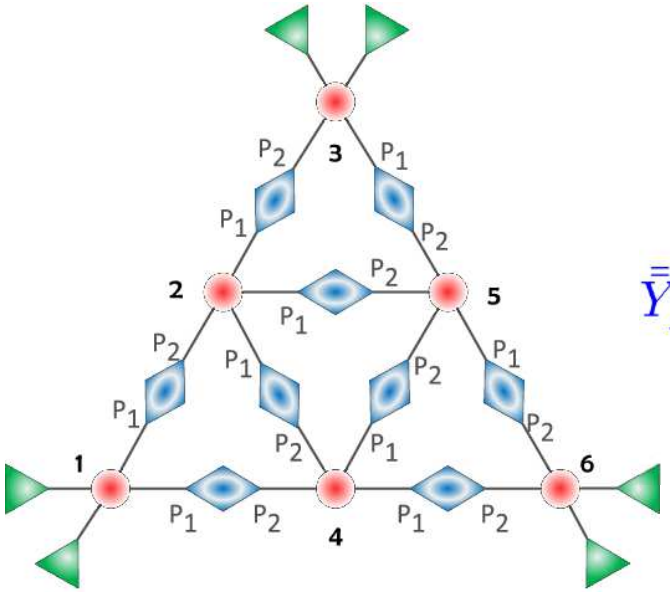


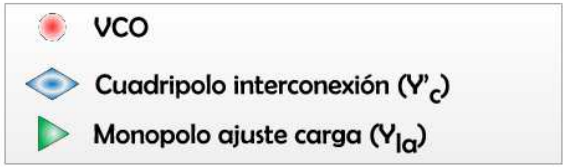


$$\bar{\bar{C}}_{gl}\Big|_{(6x6)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix} \\ \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

Redes de acoplo

- Red genérica





$$\bar{\bar{Y}}_{gl}\Big|_{(6x6)} = \begin{pmatrix} 4y_r & y_t & 0 & y_t & 0 & 0 \\ y_t & 4y_r & y_t & y_t & y_t & 0 \\ 0 & y_t & 4y_r & 0 & y_t & 0 \\ y_t & y_t & 0 & 4y_r & y_t & y_t \\ 0 & y_t & y_t & y_t & 4y_r & y_t \\ 0 & 0 & 0 & y_t & y_t & 4y_r \end{pmatrix}$$

Ajustes aplicables a la FSA

- Permitir aplicar la FSA a un mayor número de circuitos

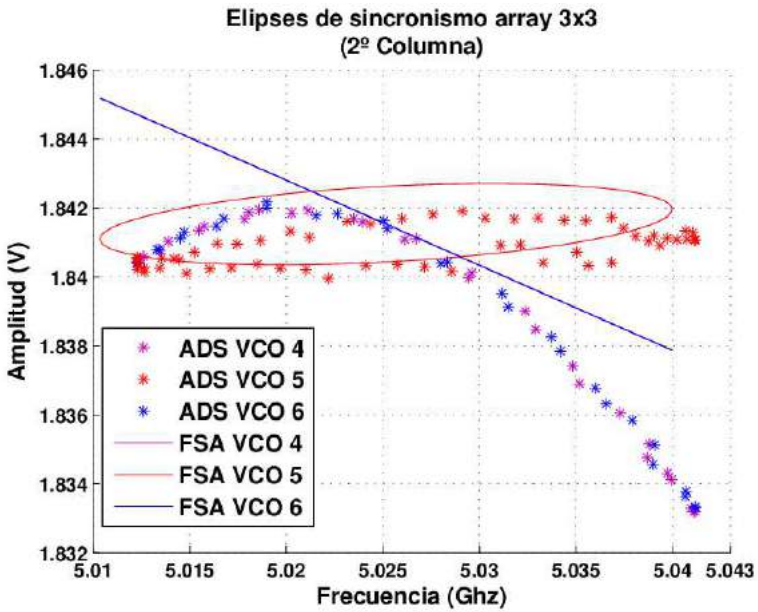
- Proporcionar una mejor comprensión de la dinámica de funcionamiento de los arrays de osciladores

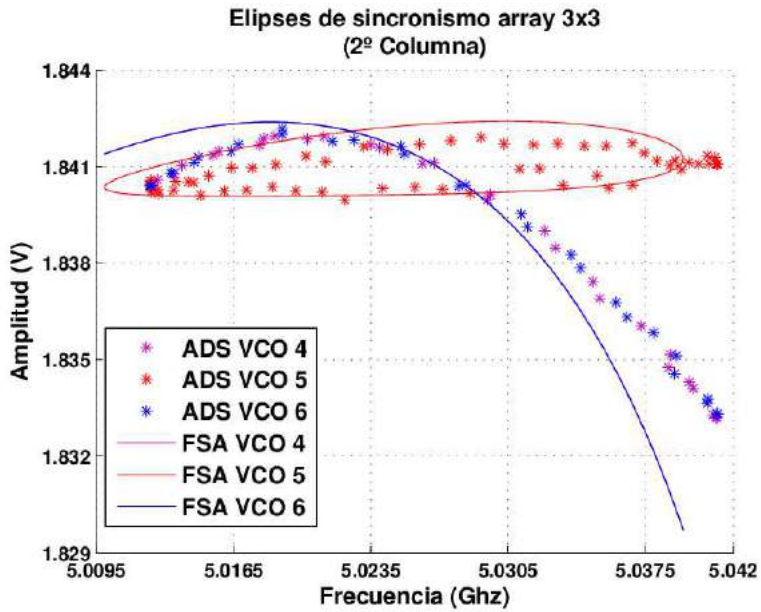
Ajustes aplicables a la FSA

- Aumento del orden de los desarrollos en serie de Taylor
  - Cálculo de las derivadas de orden hasta 2
  - Requiere modificar las expresiones del sistema para contemplar los nuevos términos

Ajustes aplicables a la FSA

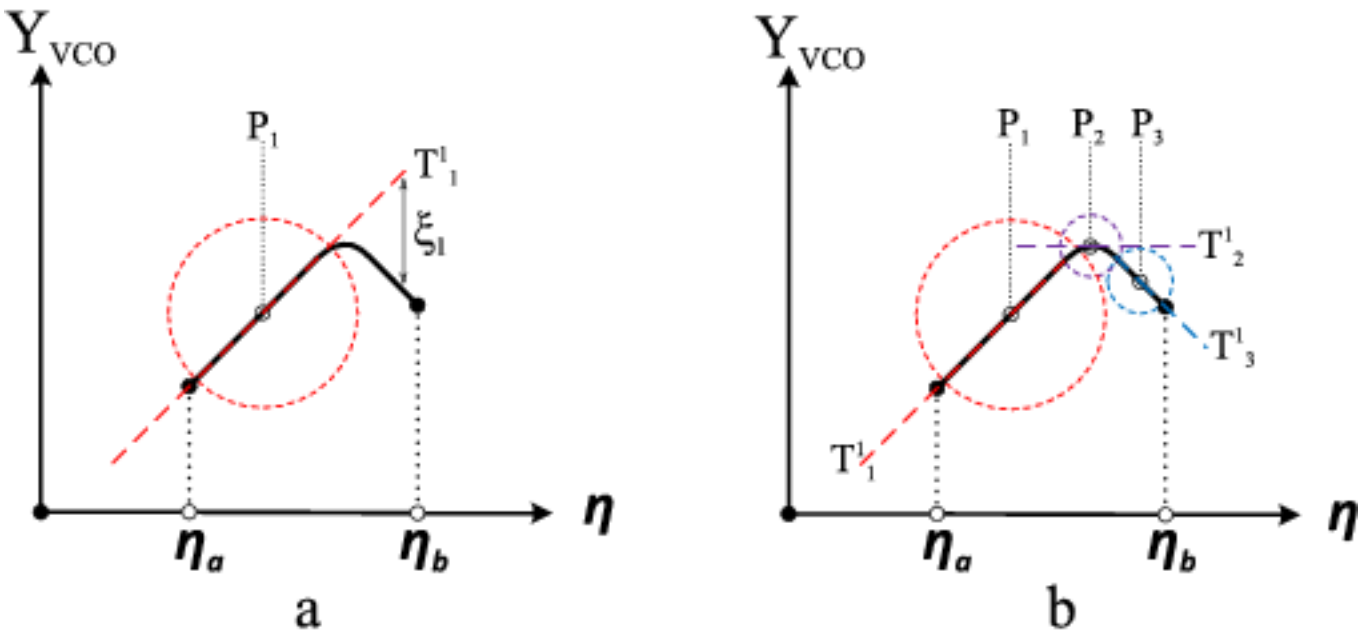
- Soluciones





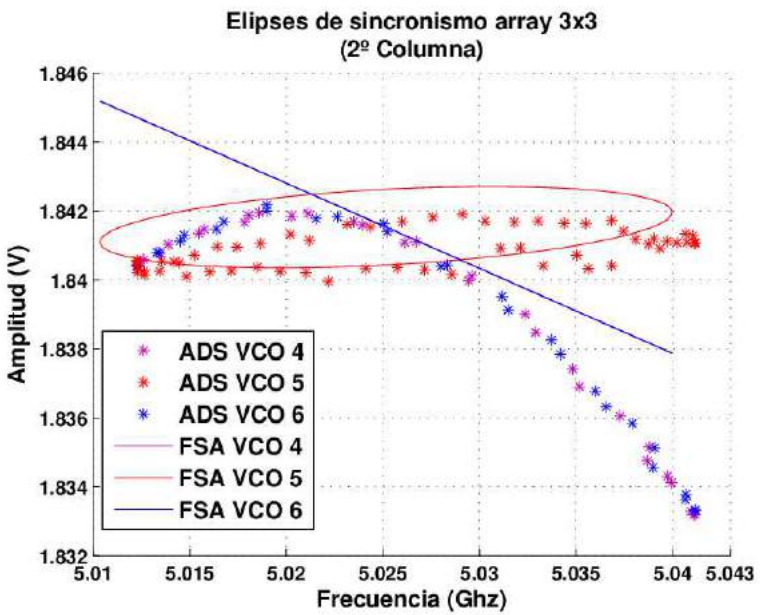
Ajustes aplicables a la FSA

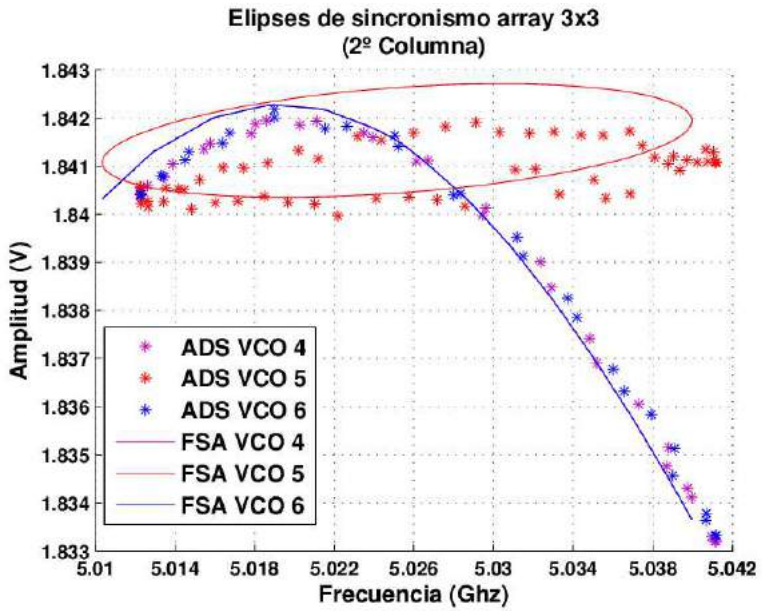
- Linealización a tramos de la respuesta de los VCOs



Ajustes aplicables a la FSA

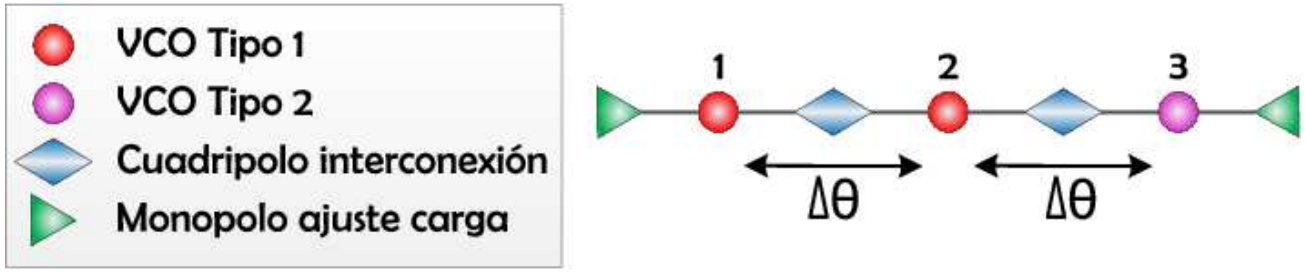
- Soluciones





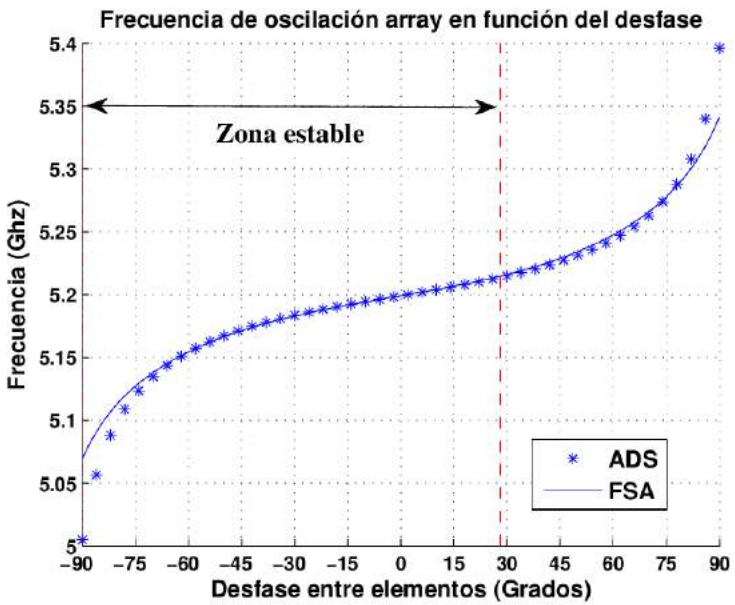
Ajustes aplicables a la FSA

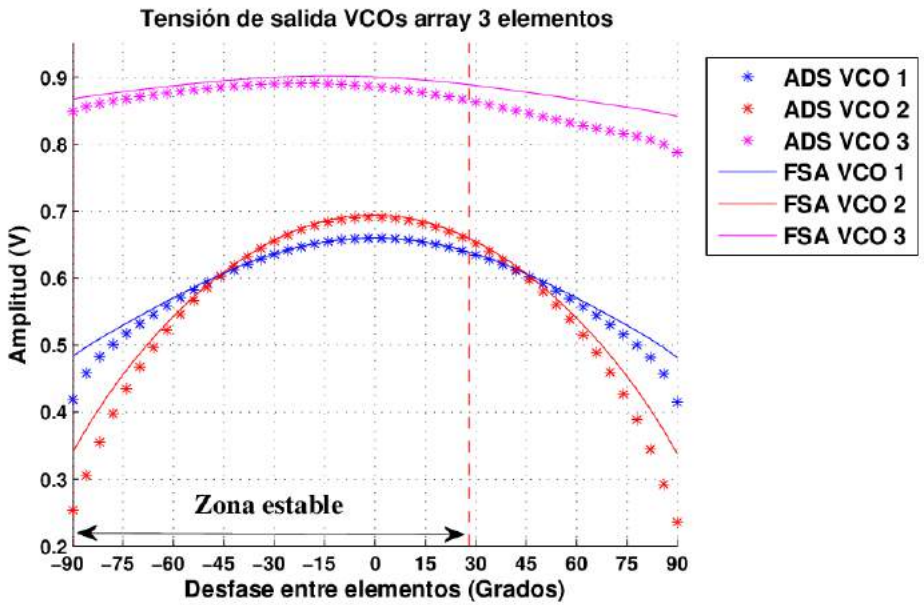
- Empleos de múltiples VCOs base
  - Cada tipo de oscilador en el sistema tiene su propio modelo (serie de Taylor)



Ajustes aplicables a la FSA

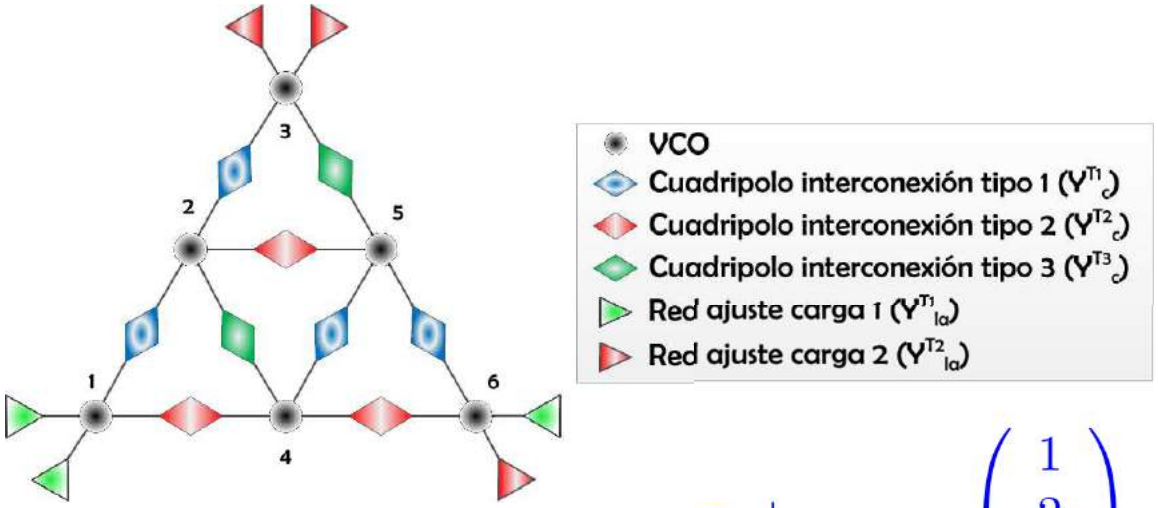
- Soluciones





Ajustes aplicables a la FSA

- Redes de acoplo de múltiples cuadripolos



$$\bar{\bar{C}}_{gl}\Big|_{(6x6)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix} \\ \begin{pmatrix} 0 & 1 & 0 & 2 & 0 & 0 \\ 1 & 0 & 1 & 3 & 2 & 0 \\ 0 & 1 & 0 & 0 & 3 & 0 \\ 2 & 3 & 0 & 0 & 1 & 2 \\ 0 & 2 & 3 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 & 1 & 0 \end{pmatrix} \end{array}$$

Ajustes aplicables a la FSA

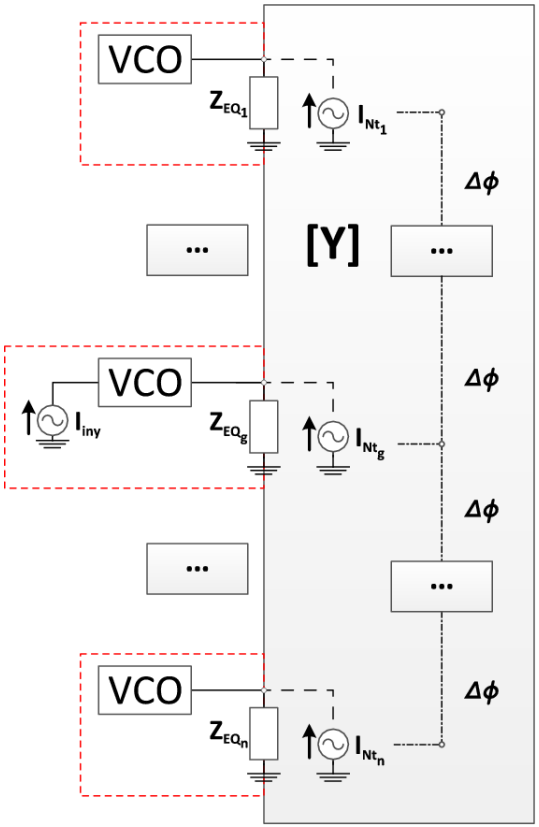
- Ajuste del modelo de los VCOs al nivel de carga equivalente

$$(Y_{VCO_i} + Y_{EQ_i})\, V_i - I_{Nt_i} = 0$$

$$Z_{EQ_i} = Y_{EQ_i}^{-1} = (Y_{L_i} + y_{ii})^{-1}$$

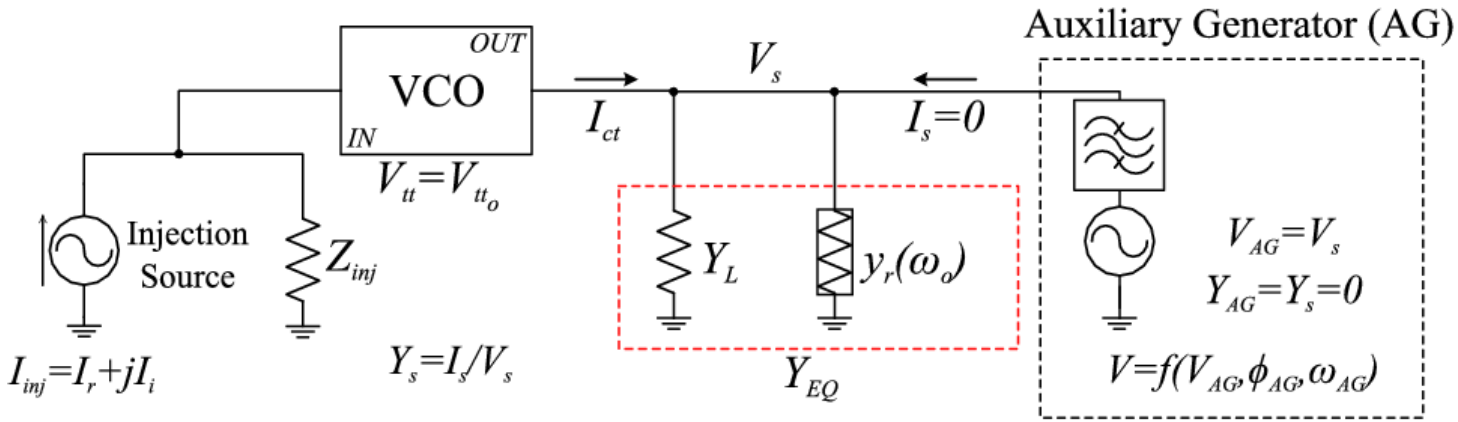
$$I_{Nt_i} = -\sum_{\substack{k=1 \\ k \neq i}}^{n} V_k y_{ik}$$

$$(i = 1 \cdots n)$$



Ajustes aplicables a la FSA

- Se obtienen las derivadas sobre Y_{EQ}

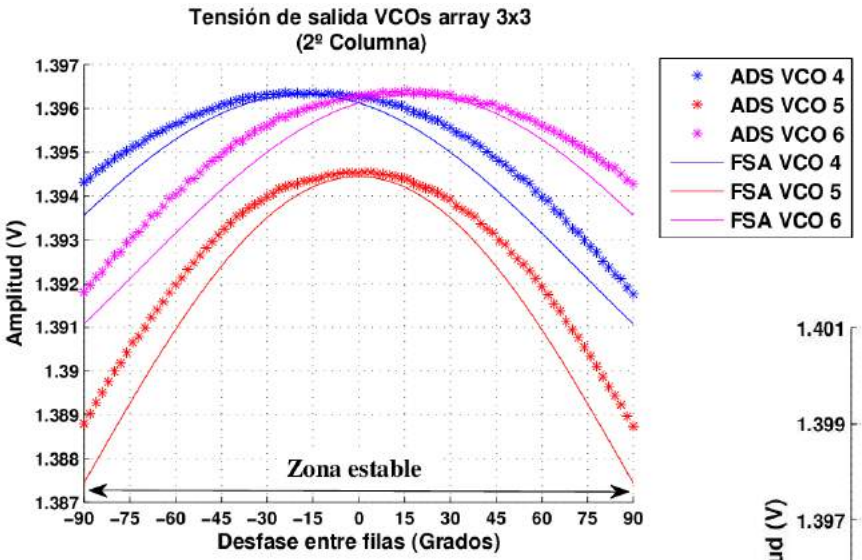


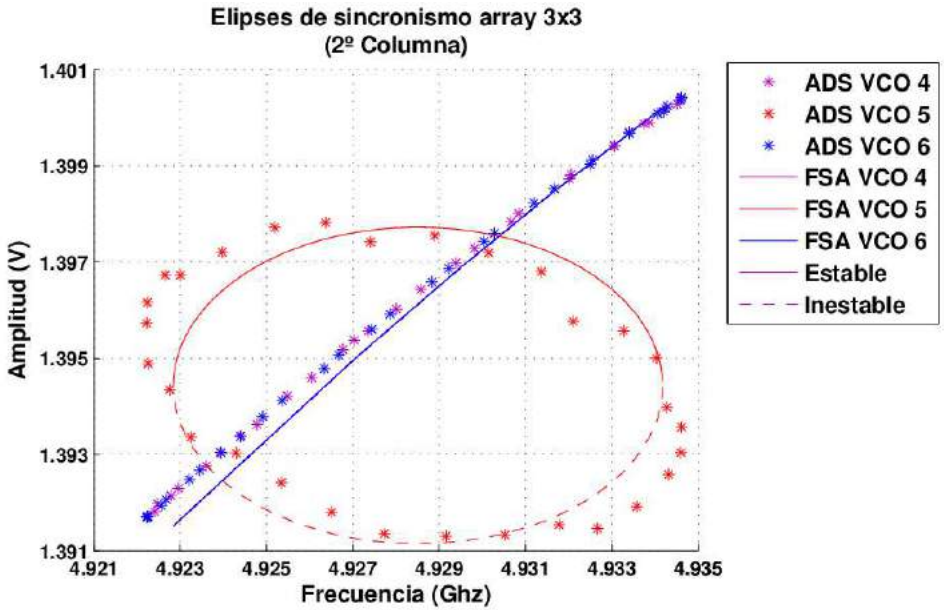
$$(Y_{VCO}+Y_L)\approx -y_r(\omega_o)+\frac{\delta Y}{\delta V}\Big|_{Z_{EQ}}\left(V_g-V_o|_{Z_{EQ}}\right)+\frac{\delta Y}{\delta \omega}\Big|_{Z_{EQ}}\left(\omega_{iny}-\omega_o|_{Z_{EQ}}\right)+\frac{\delta Y}{\delta \eta}\Big|_{Z_{EQ}}(\eta_g-\eta_o)+$$

$$+I_{iny}\left(\frac{\delta Y}{\delta I^r}\Big|_{Z_{EQ}}\cos(\phi_{iny}-\phi_g)+\frac{\delta Y}{\delta I^i}\Big|_{Z_{EQ}}\sin(\phi_{iny}-\phi_g)\right)$$

Ajustes aplicables a la FSA

- Soluciones





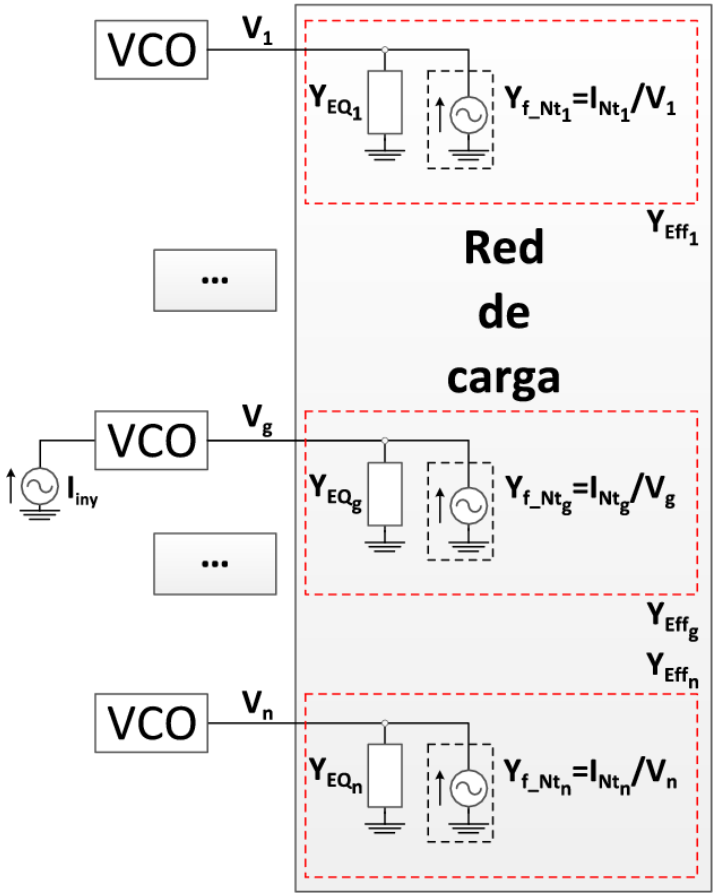
Ajustes aplicables a la FSA

- Acoplo fuerte

$$Y_{VCO_i} + Y_{Eff_i} = 0$$

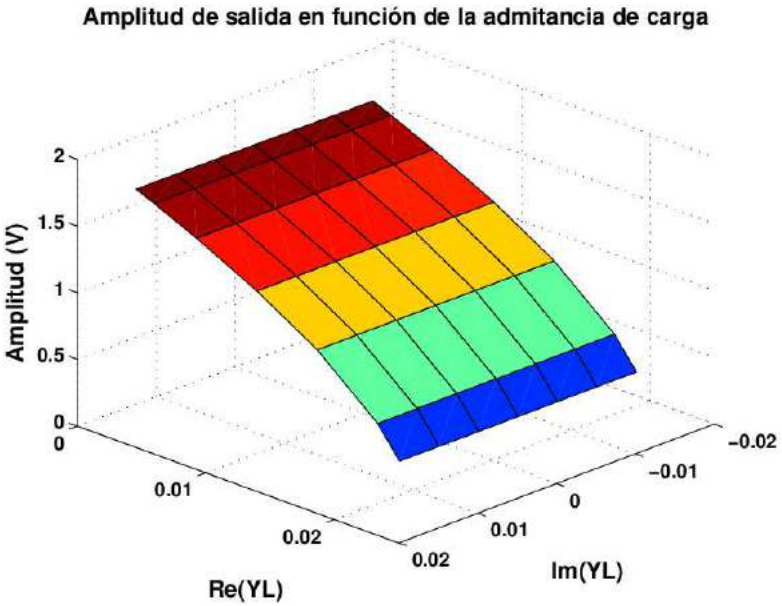
$$Y_{Eff_i} = Y_{EQ_i} + Y_{f_Nt_i}$$

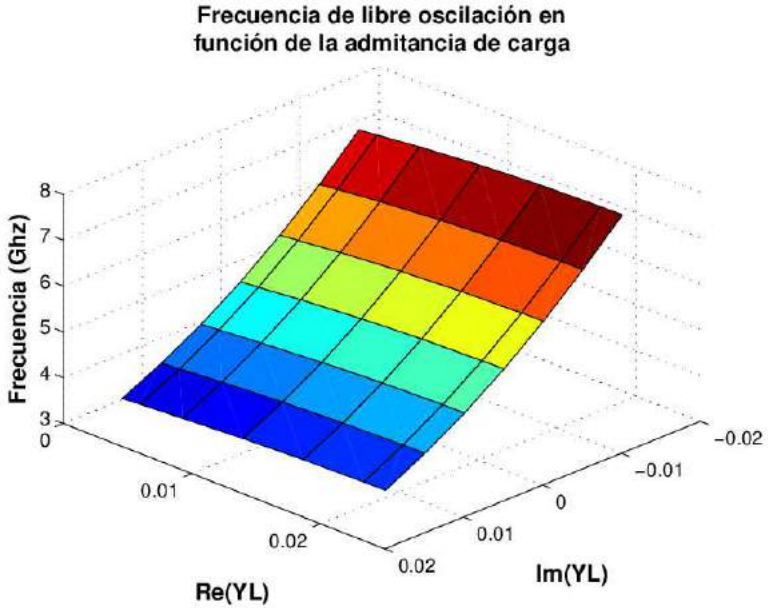
$$(i = 1 \cdots n)$$



Ajustes aplicables a la FSA

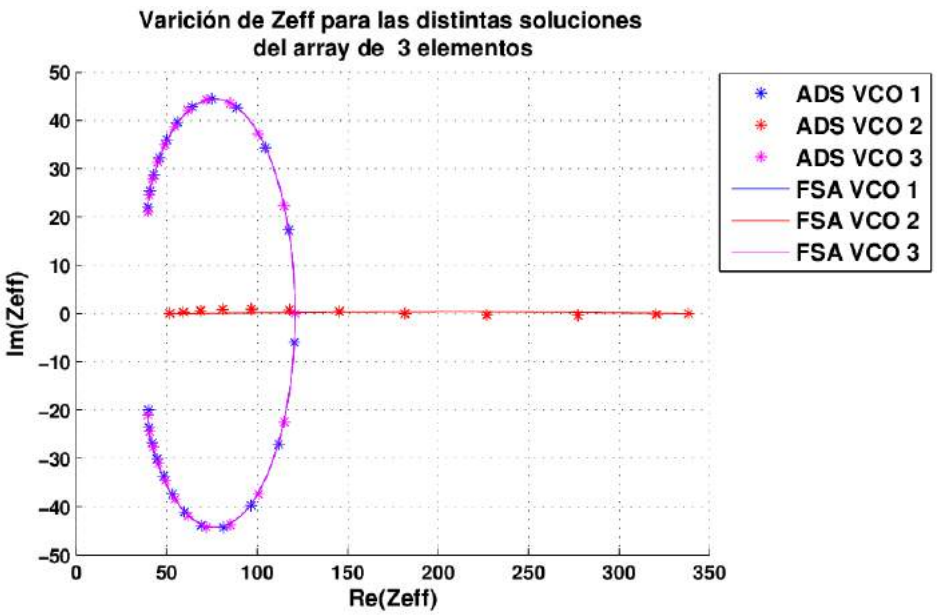
- Se modela la respuesta de los VCOs en un volumen de admitancias

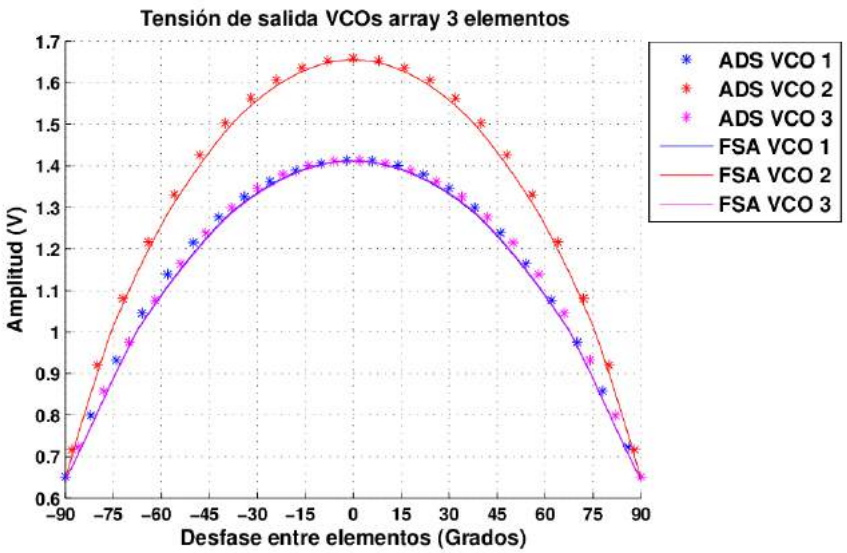




Ajustes aplicables a la FSA

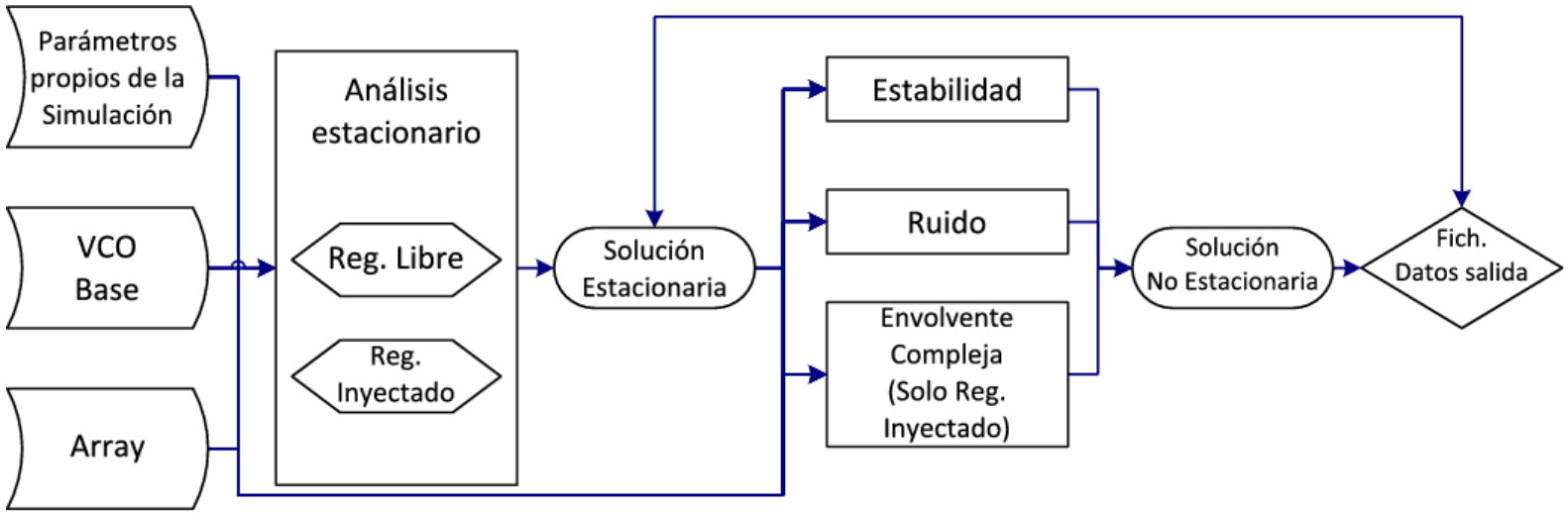
- Soluciones





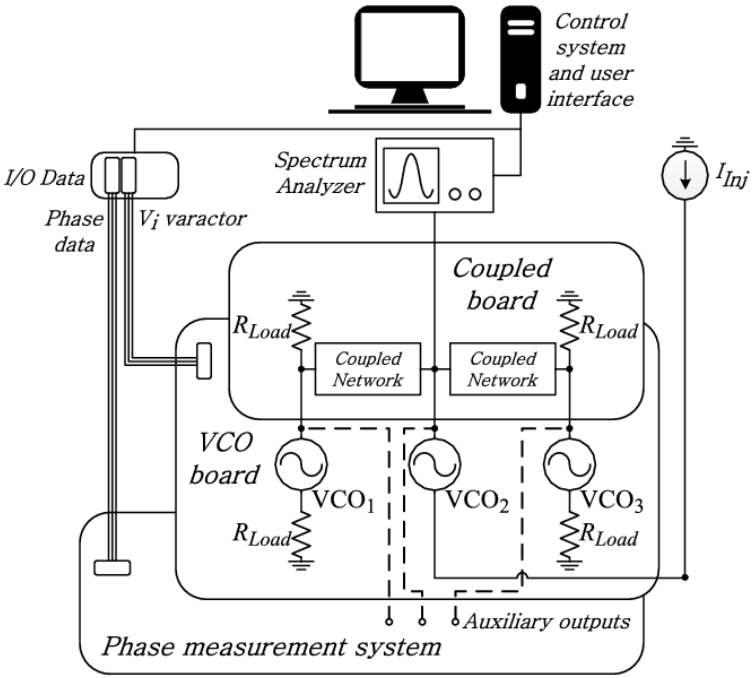
Aplicación práctica

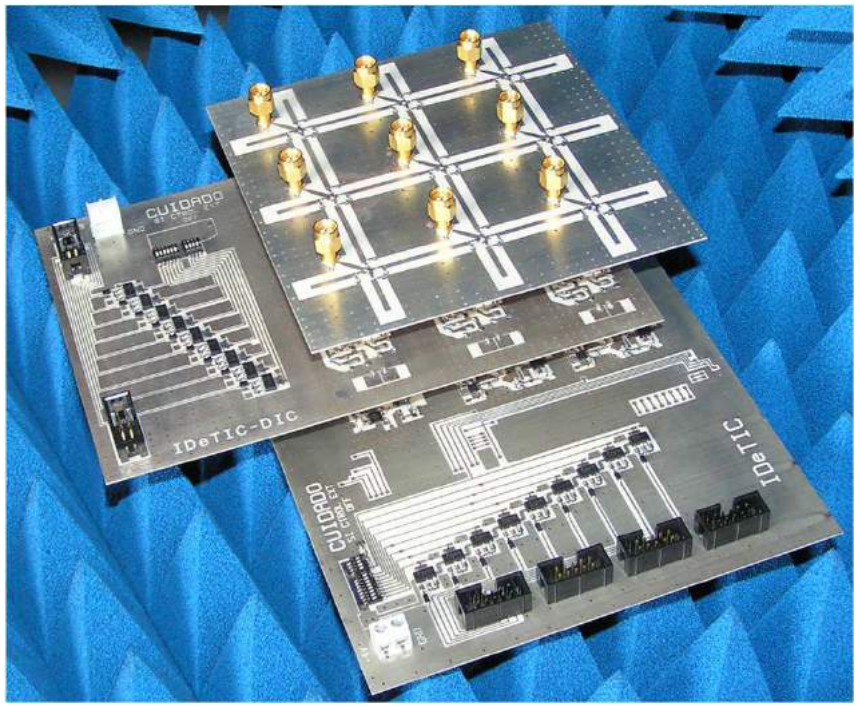
- Implementación Matlab



Aplicación práctica

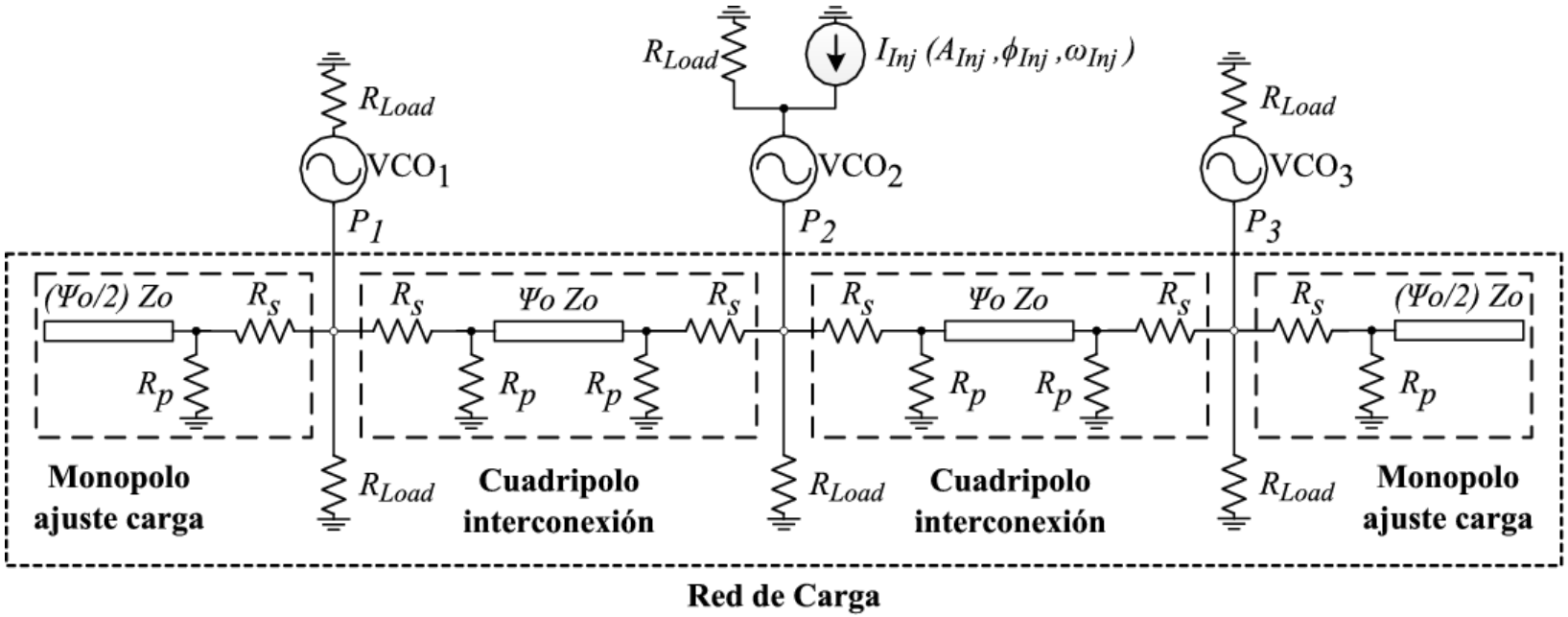
- Esquema del sistema





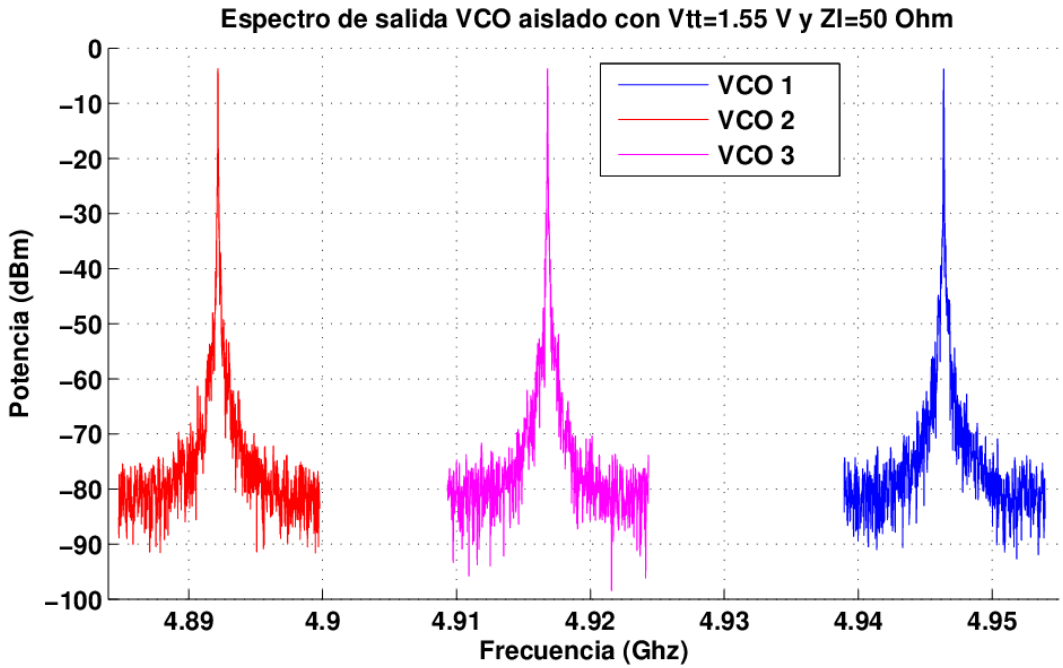
Aplicación práctica

- Array lineal $3x1$



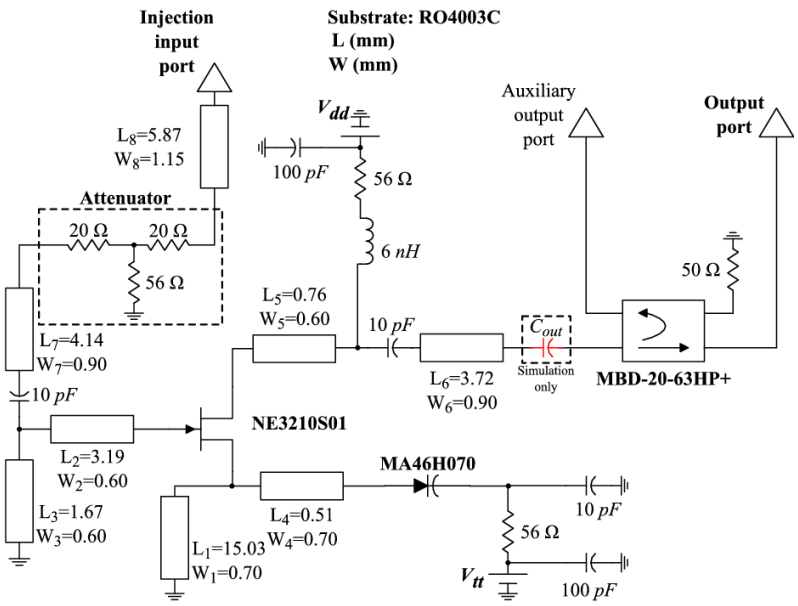
Aplicación práctica

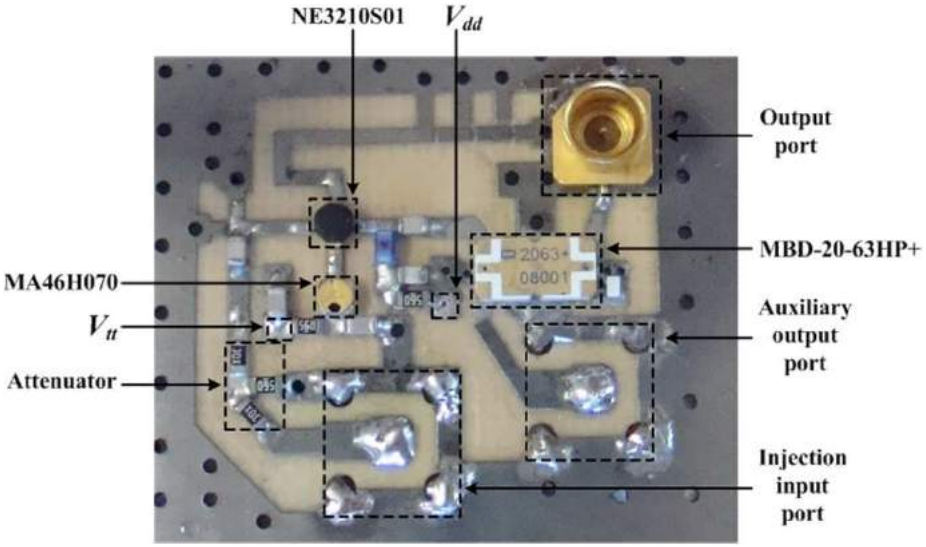
- Soluciones estacionarias
  - Medida de osciladores aislados



Aplicación práctica

- Modelo de los VCOs

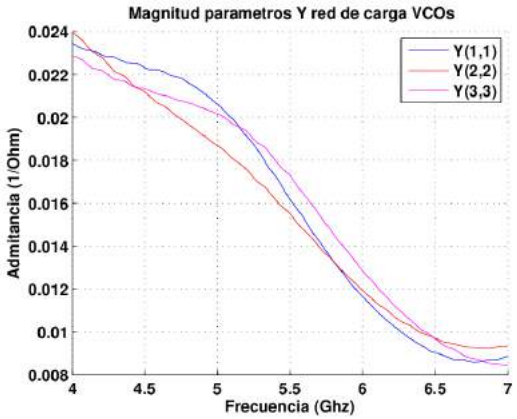


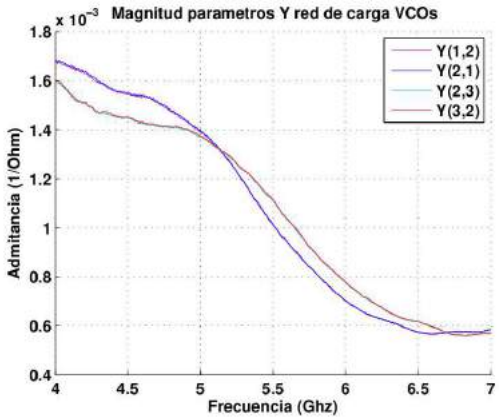


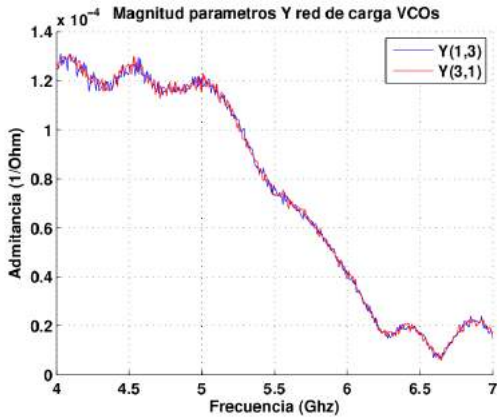
$$C_{out_1} = 3\,pF \quad -- \quad C_{out_3} = 4\,pF$$

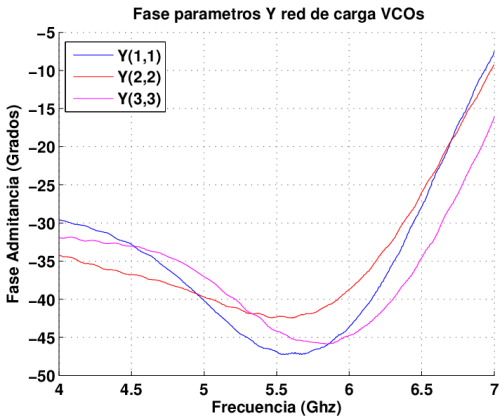
Aplicación práctica

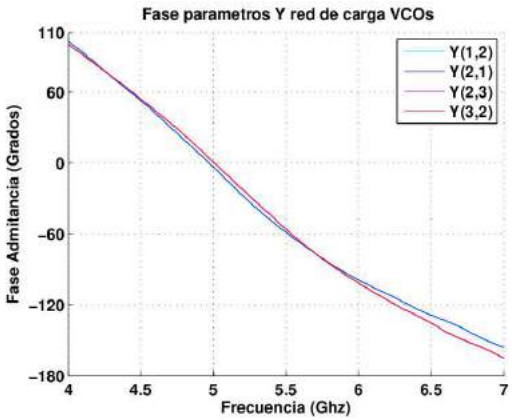
- Parámetros Y red de acoplo

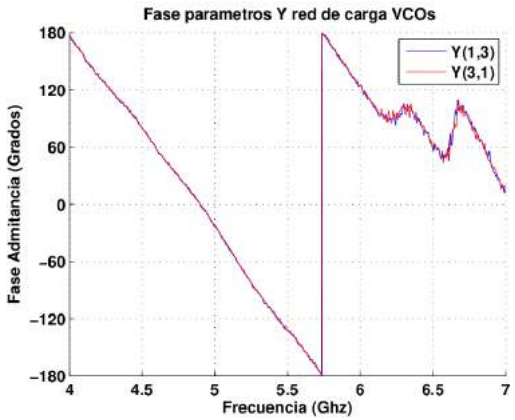






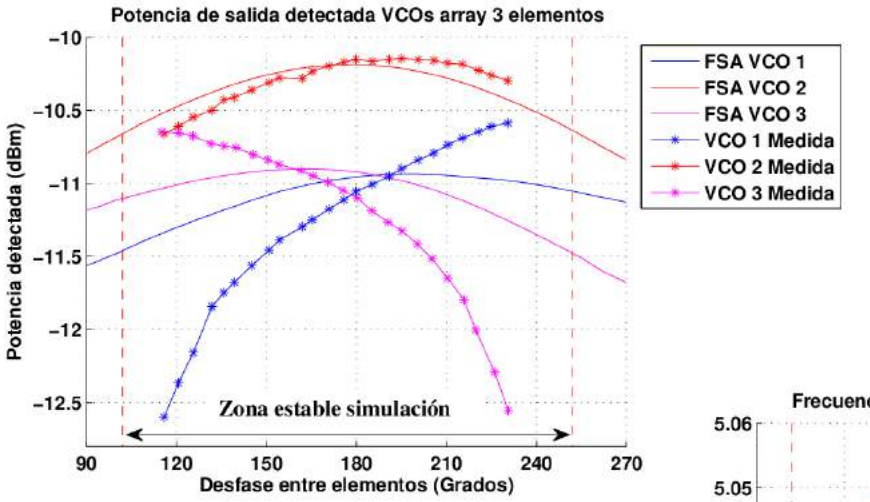


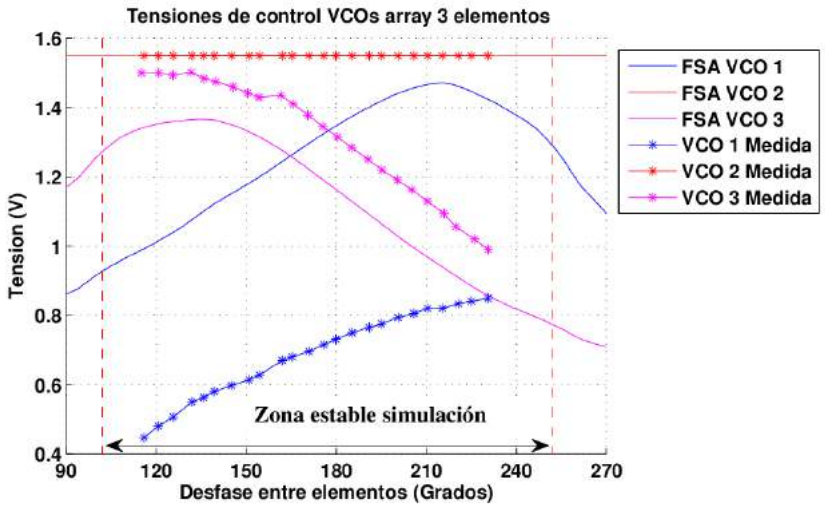


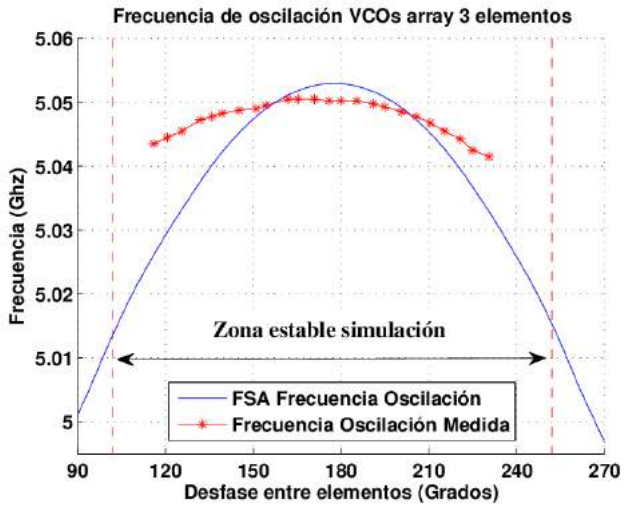


Aplicación práctica

- Régimen libre

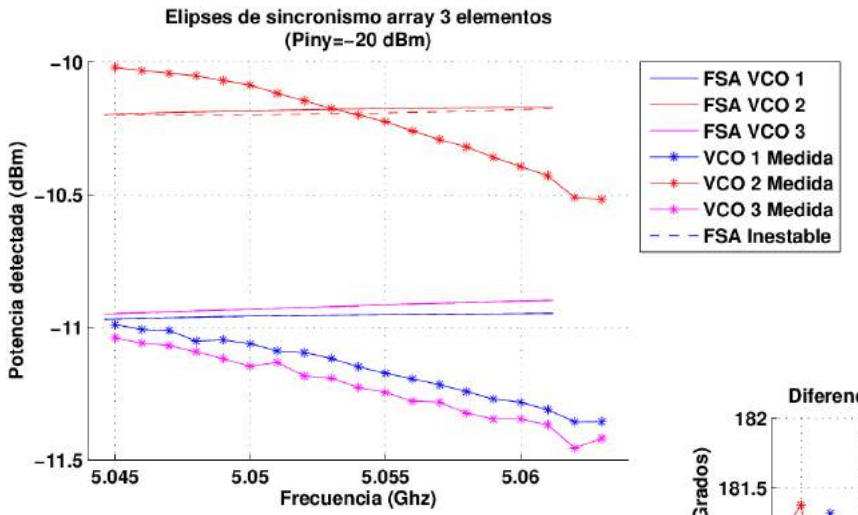


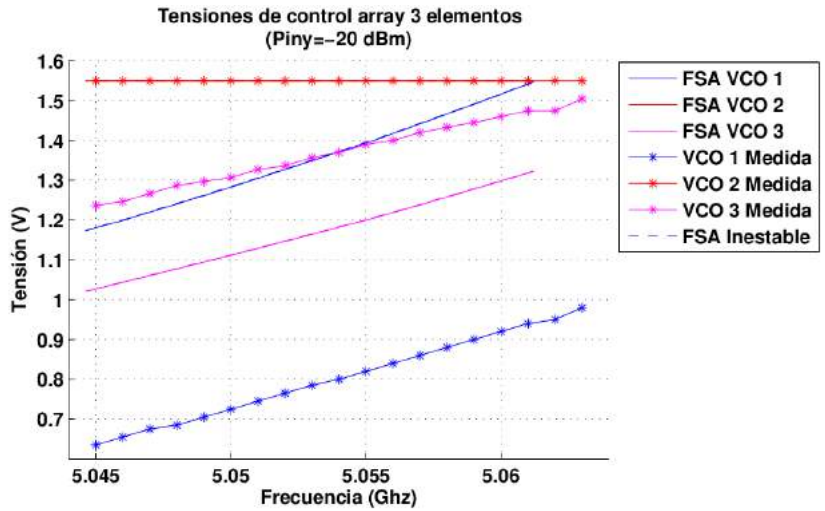


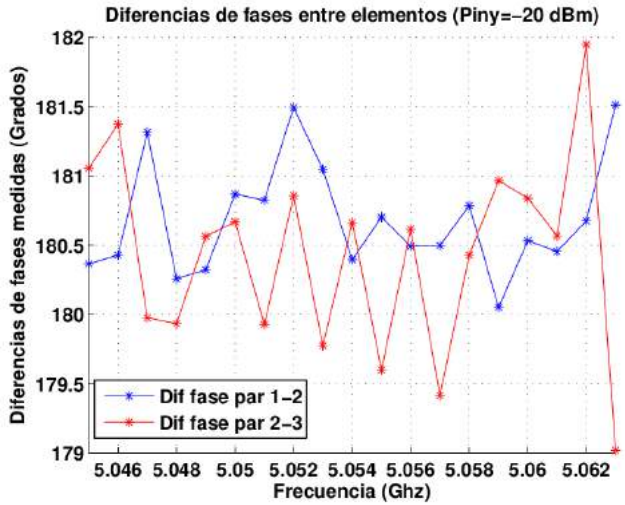


Aplicación práctica

- Régimen inyectado

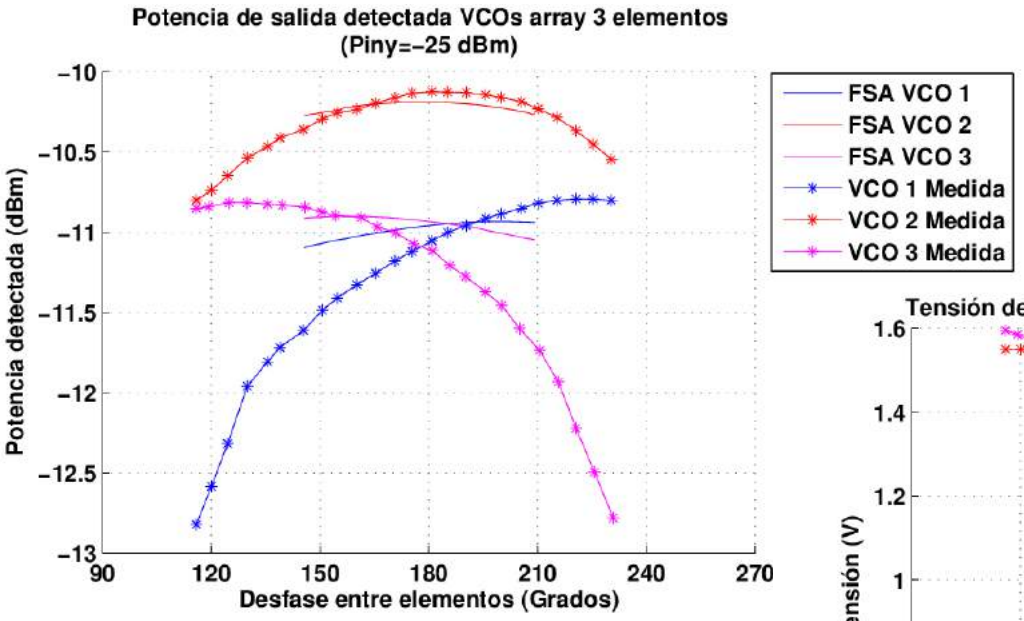


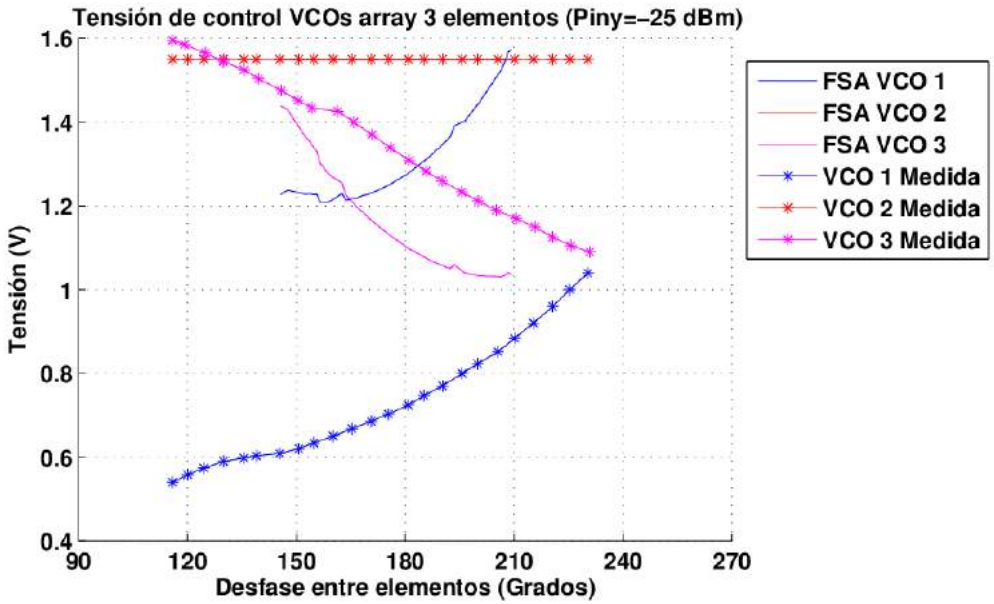




Aplicación práctica

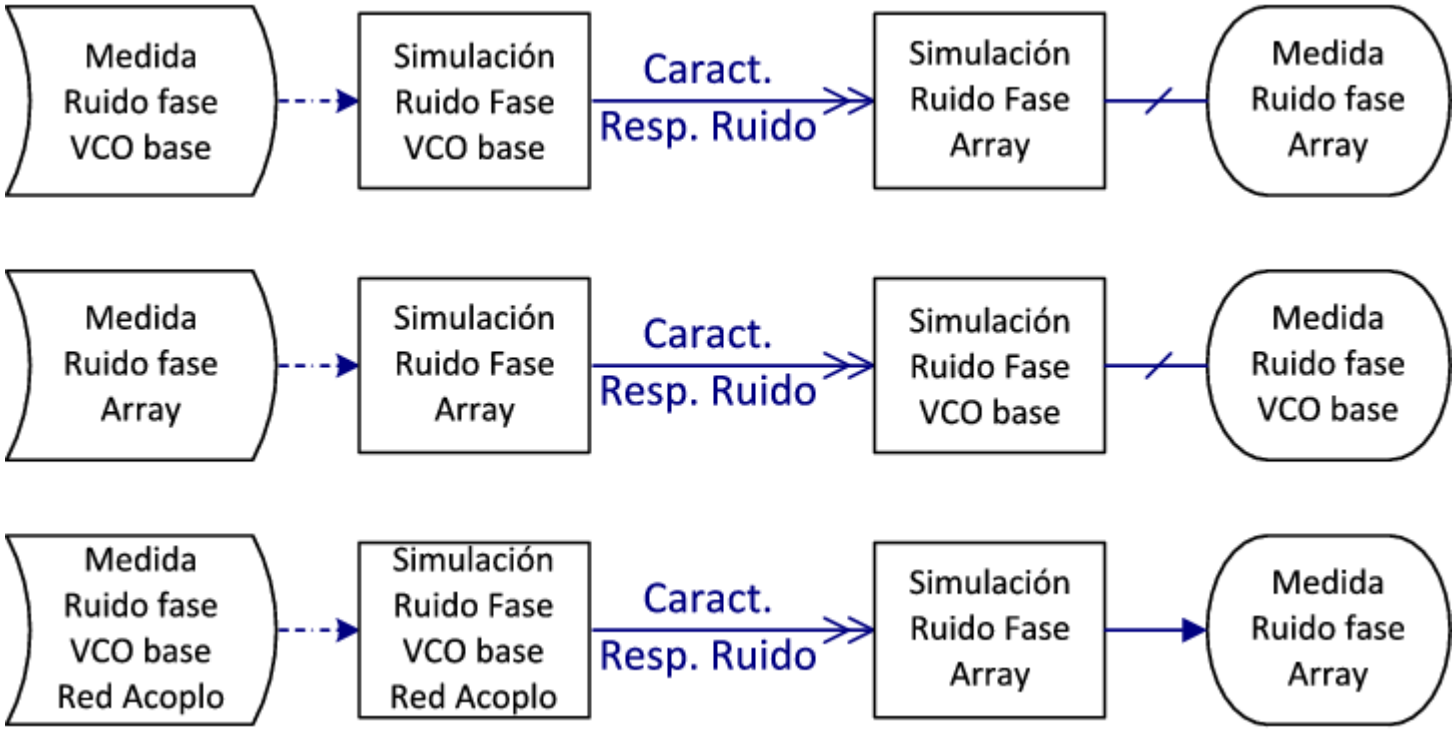
- Régimen inyectado





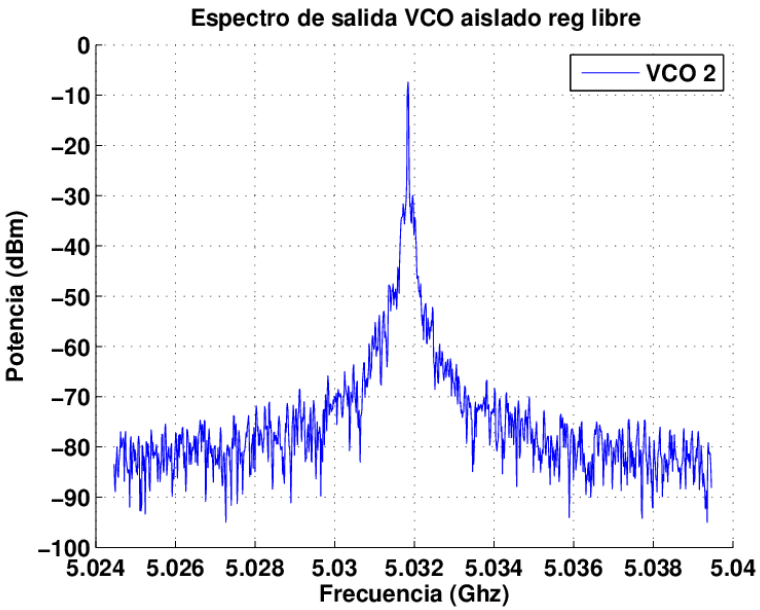
Aplicación práctica

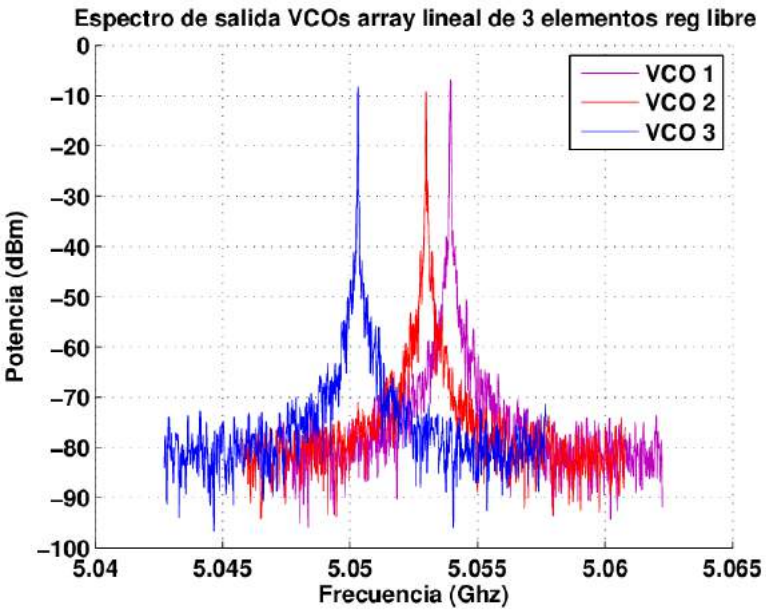
- Ruido de fase



Aplicación práctica

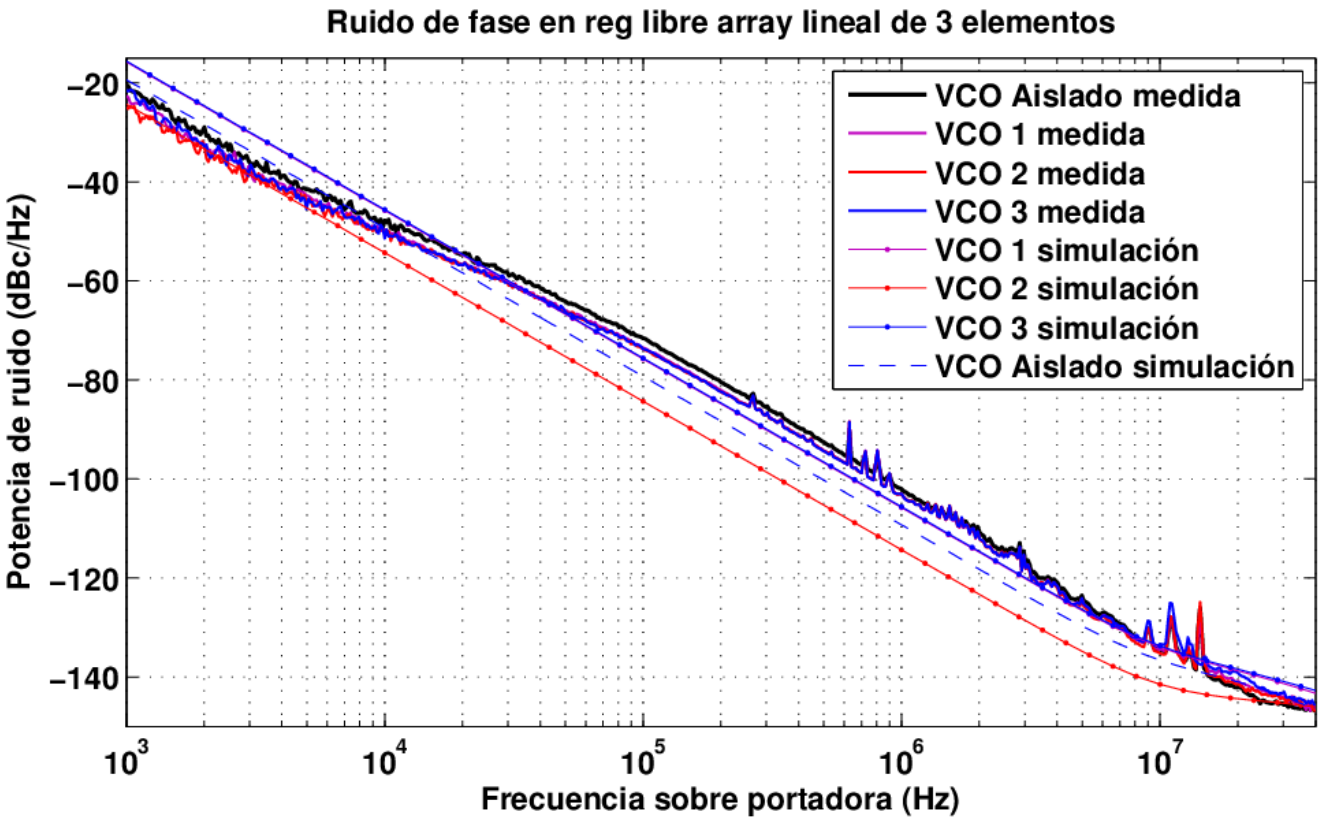
- Régimen libre





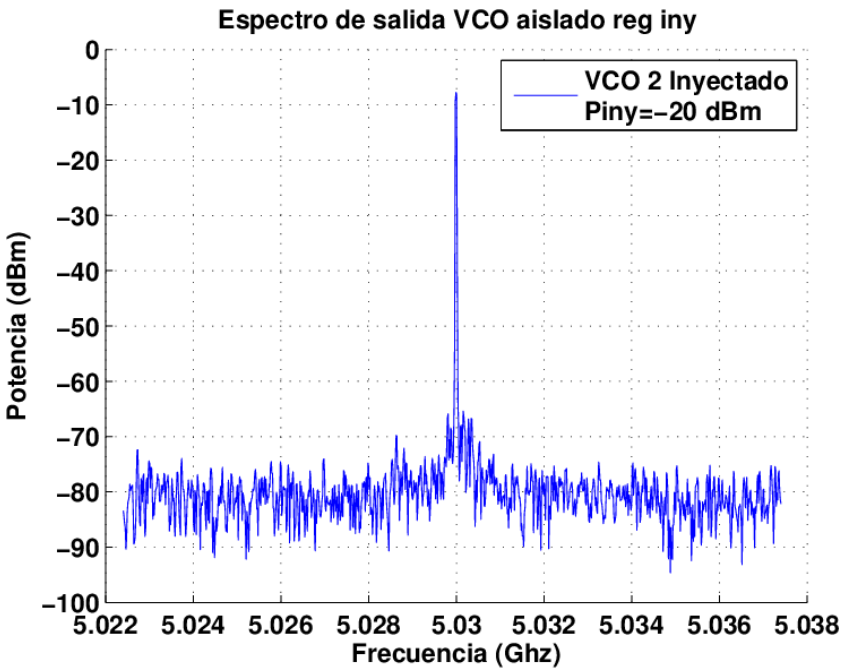
Aplicación práctica

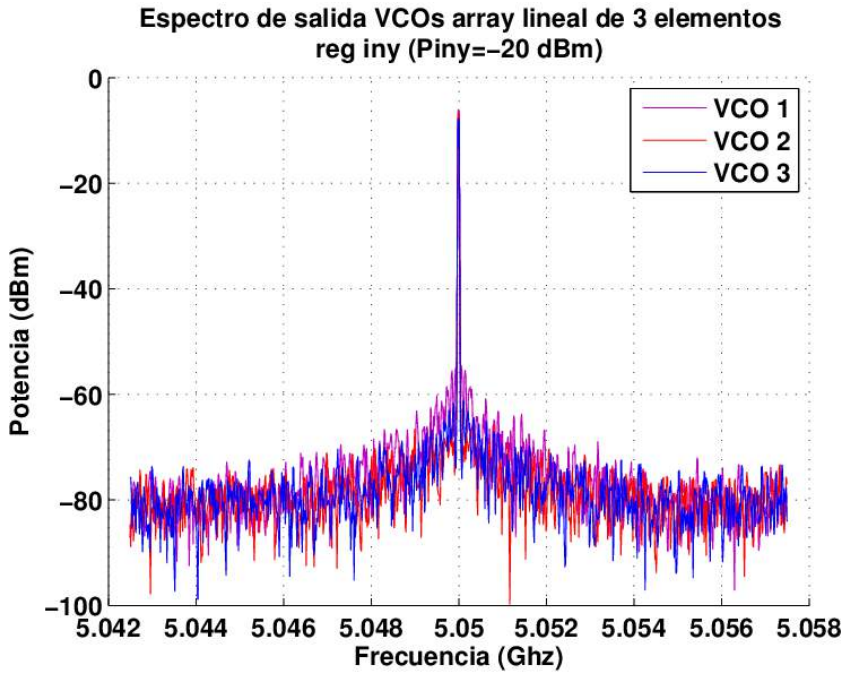
- Régimen libre



Aplicación práctica

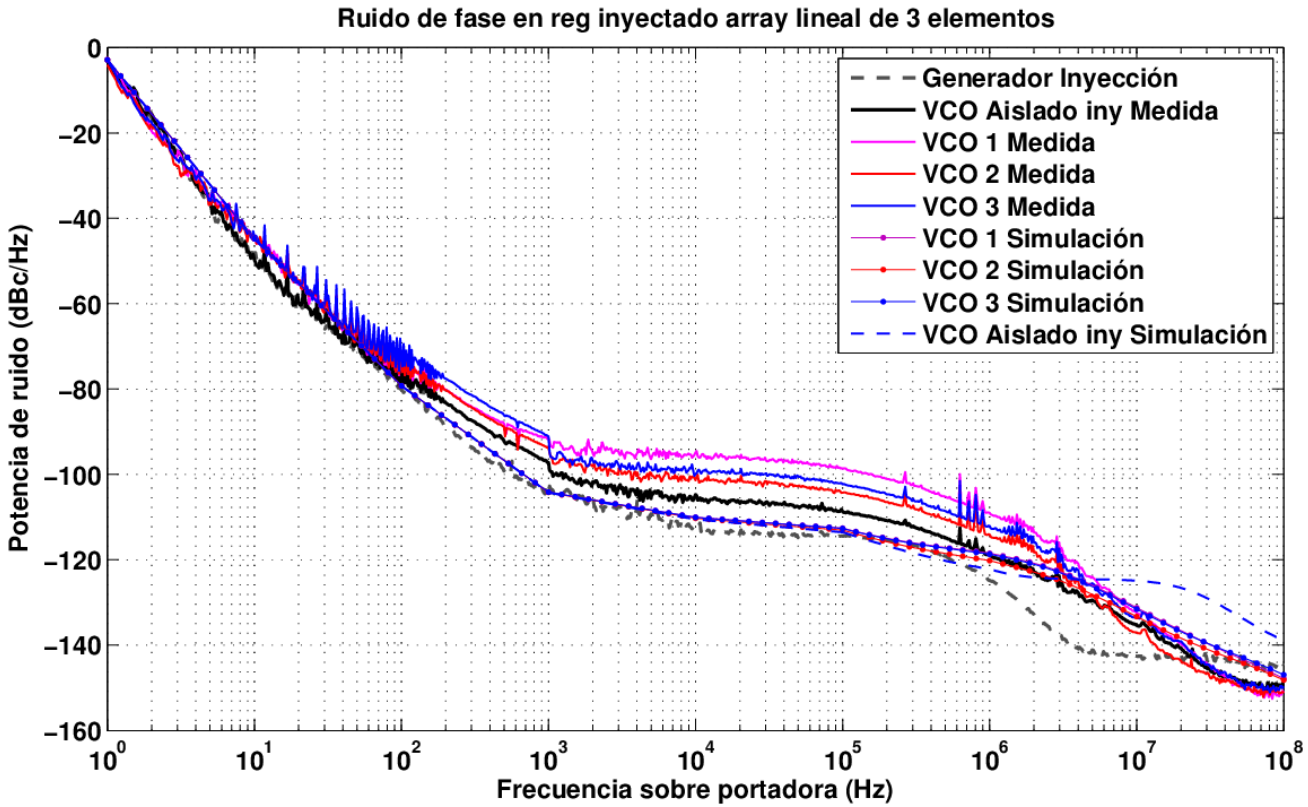
- Régimen inyectado





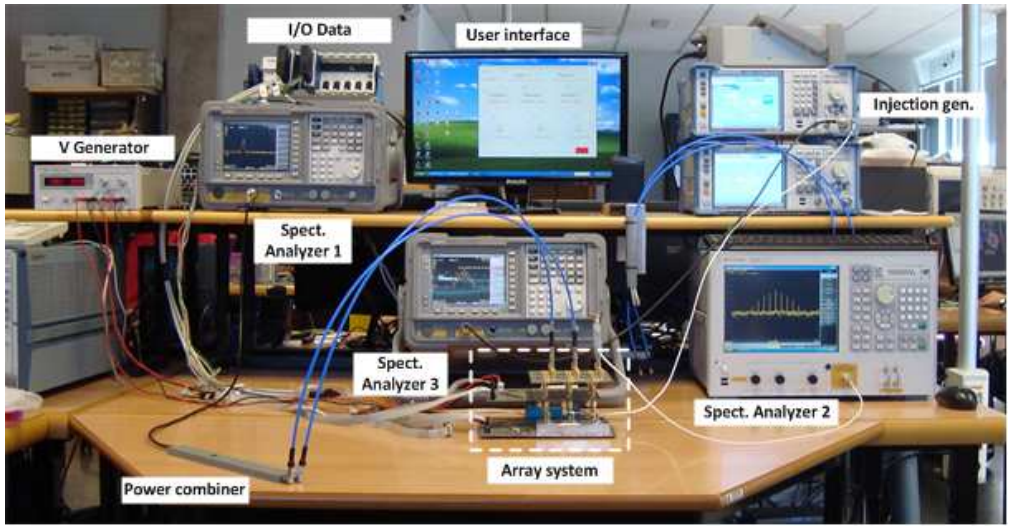
Aplicación práctica

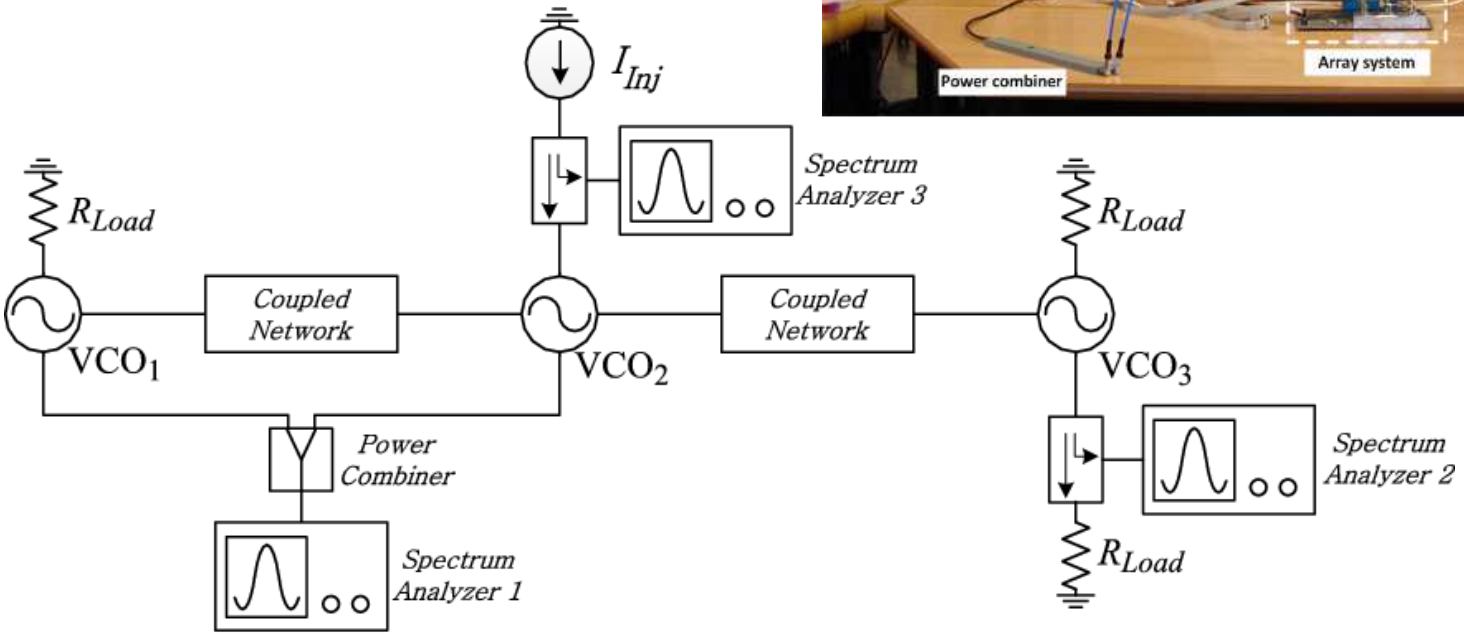
- Régimen inyectado



Aplicación práctica

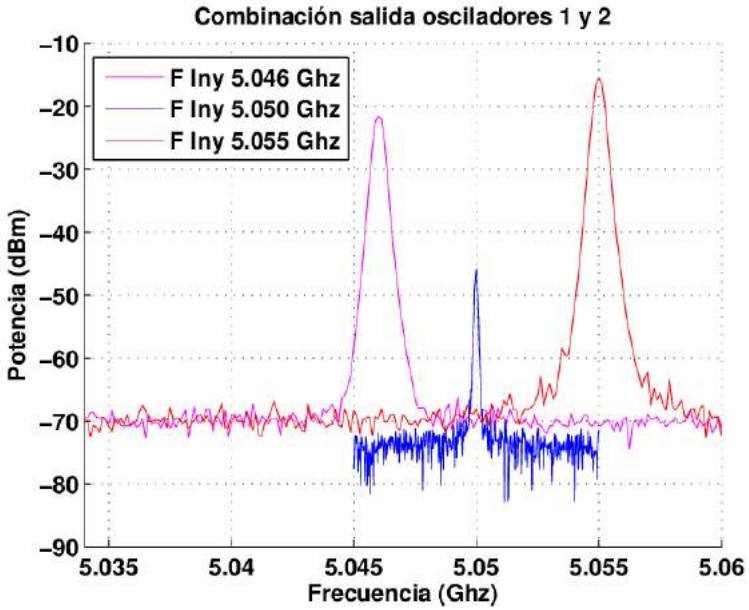
- Inyección de señales moduladas

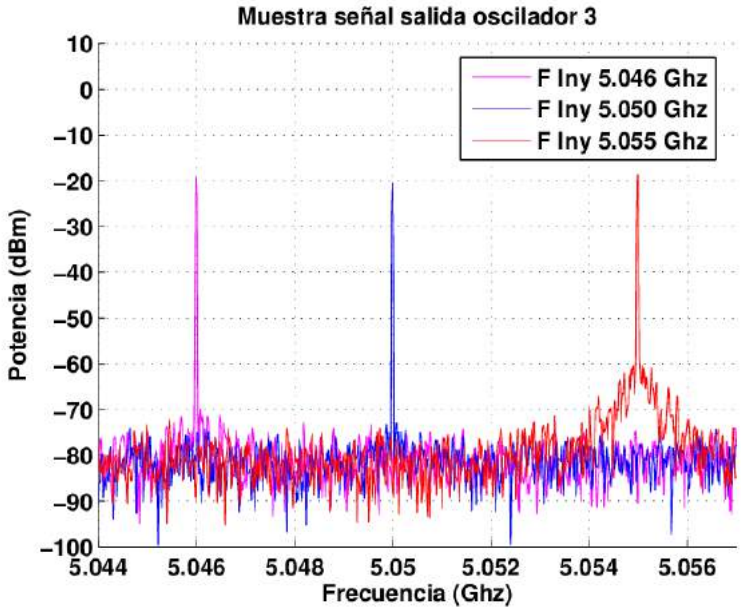


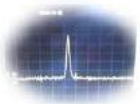


Aplicación práctica

- Soluciones

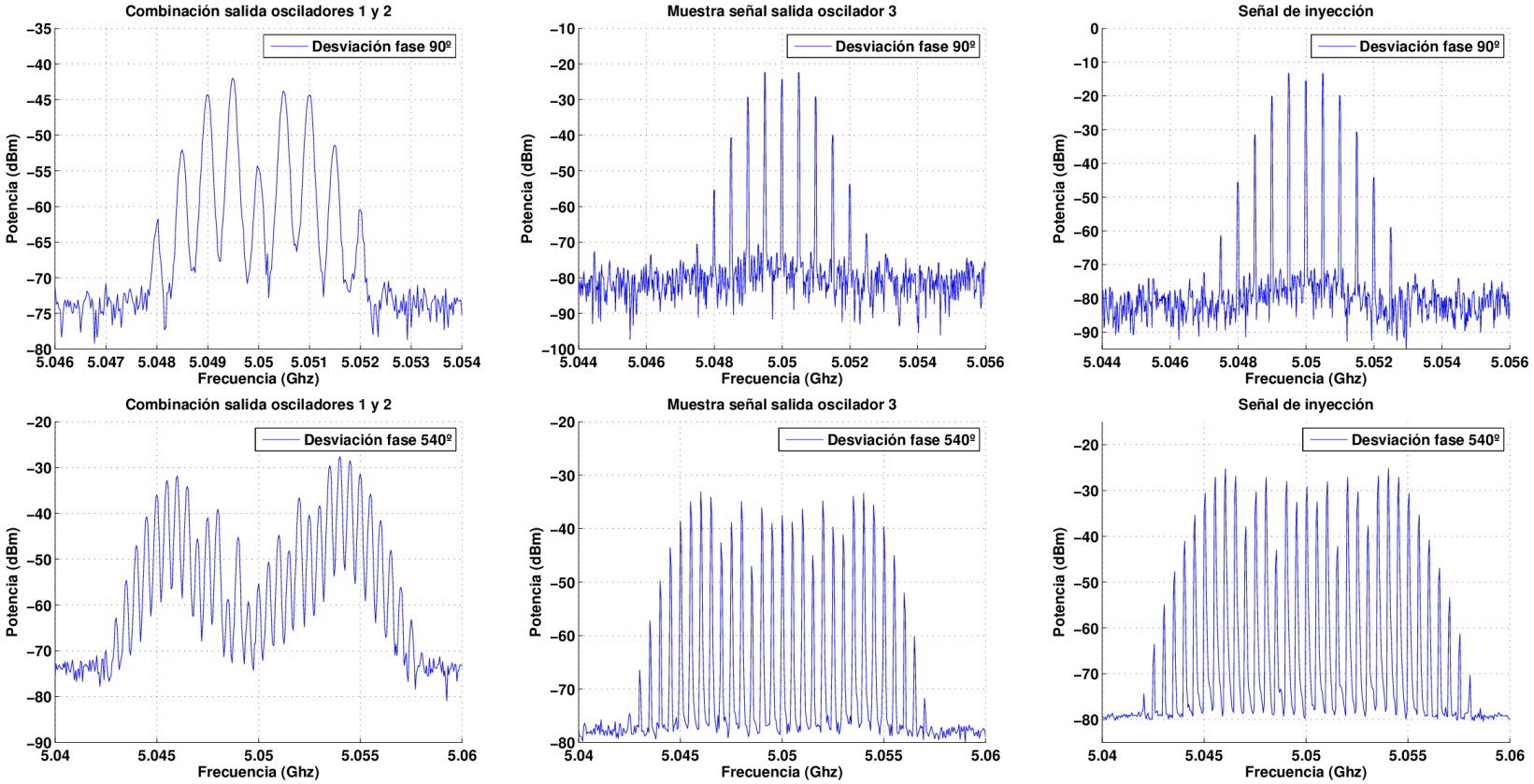






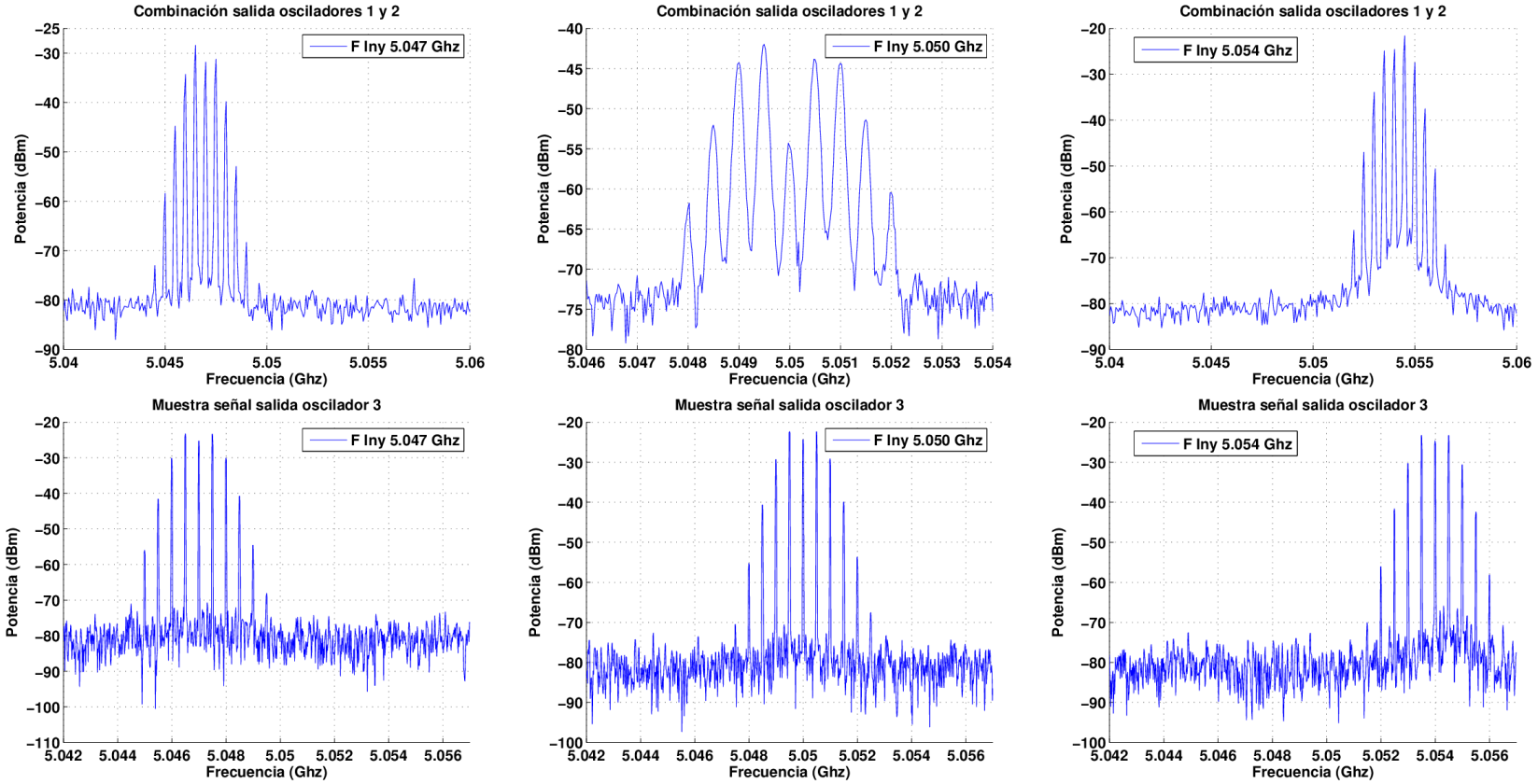
Aplicación práctica

- Soluciones



Aplicación práctica

- Soluciones



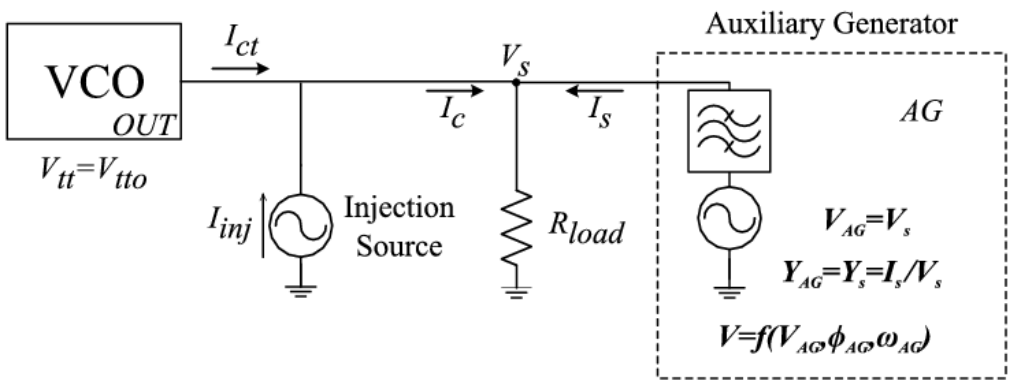
Medida de las derivadas

- Obtención de las derivadas a partir de datos medidos en vez de simulados

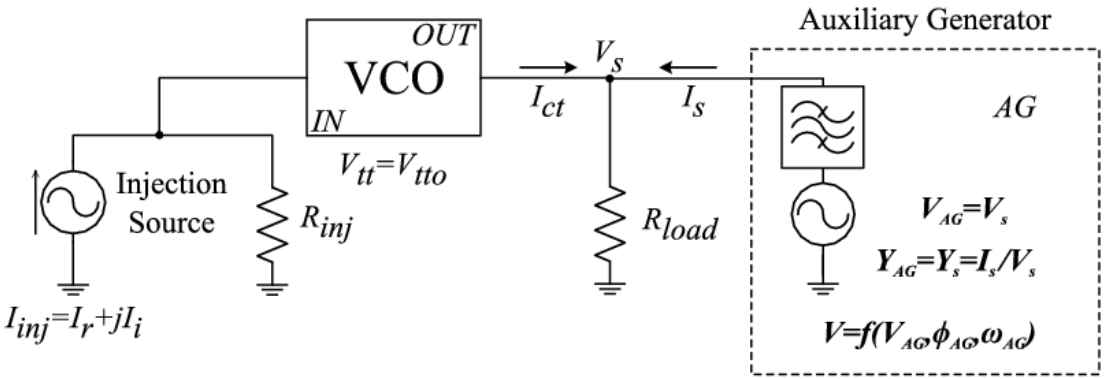
- Cálculo a partir de soluciones sincronizadas

Medida de las derivadas

- Sistema empleado



(a)



(b)

Medida de las derivadas

- Cálculo derivadas régimen libre

$$Y_{s1} - Y_{s2} \simeq \frac{\delta Y_s}{\delta V}\left(V_{s1} - V_{s2}\right) + \frac{\delta Y_s}{\delta \omega}\left(\omega_{s1} - \omega_{s2}\right) + \frac{\delta Y_s}{\delta \eta}\left(\eta_1 - \eta_2\right)$$

$$\frac{\delta Y_s}{\delta V} \simeq \frac{Y_{s1} - Y_{s2}}{V_{s1} - V_{s2}}$$

$$\frac{\delta Y_s}{\delta \omega} \simeq \frac{Y'_{s1} - Y'_{s2} - \frac{\delta Y_s}{\delta V}\left(V'_{s1} - V'_{s2}\right)}{\omega'_{s1} - \omega'_{s2}}$$

$$\frac{\delta Y_s}{\delta \eta} \simeq \frac{Y_{s1}^{\eta 1} - Y_{s2}^{\eta 2} - \frac{\delta Y_s}{\delta V}\left(V_{s1}^{\eta 1} - V_{s2}^{\eta 2}\right) - \frac{\delta Y_s}{\delta \omega}\left(\omega_{s1}^{\eta 1} - \omega_{s2}^{\eta 2}\right)}{\eta_1 - \eta_2}$$

Medida de las derivadas

- Cálculo derivadas régimen inyectado

$$\frac{\delta Y_s}{\delta V}\left(V_{sA}-V_{sB}\right)+\frac{\delta Y_s}{\delta \omega}\left(\omega_{sA}-\omega_{sB}\right)=$$

$$=\frac{\delta Y_s}{\delta I^r_{inj}}\left(I^B_{inj}\cos\left(\Delta\phi_B\right)-I^A_{inj}\cos\left(\Delta\phi_A\right)\right)+\frac{\delta Y_s}{\delta I^i_{inj}}\left(I^B_{inj}\sin\left(\Delta\phi_B\right)-I^A_{inj}\sin\left(\Delta\phi_A\right)\right)$$

$$\frac{\delta Y_s}{\delta I^r_{inj}}\simeq\frac{C_1\beta_2-C_2\beta_1}{\alpha_1\beta_2-\alpha_2\beta_1}\qquad\frac{\delta Y_s}{\delta I^i_{inj}}\simeq\frac{\alpha_1C_2-\alpha_2C_1}{\alpha_1\beta_2-\alpha_2\beta_1}$$

$$C_1=\frac{\delta Y_s}{\delta V}\left(V_{s1}-V_{s2}\right)+\frac{\delta Y_s}{\delta \omega}\left(\omega_{s1}-\omega_{s2}\right)$$

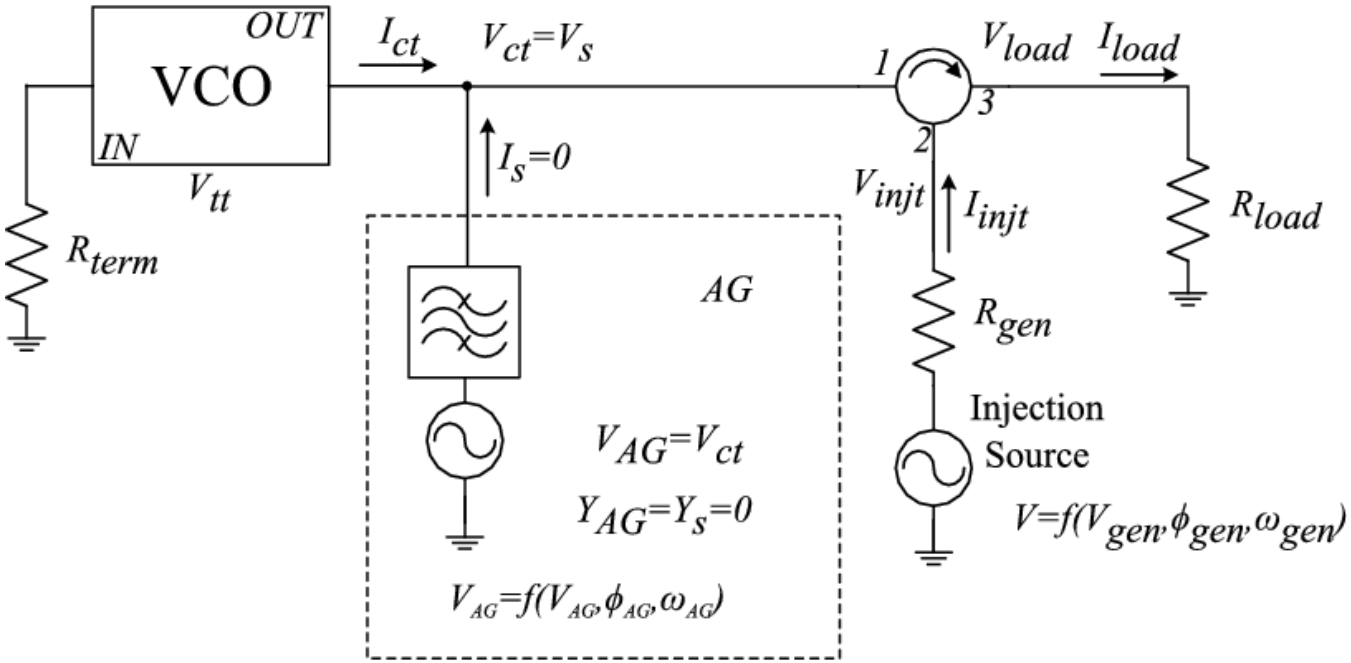
$$C_2=\frac{\delta Y_s}{\delta V}\left(V_{s3}-V_{s4}\right)+\frac{\delta Y_s}{\delta \omega}\left(\omega_{s3}-\omega_{s4}\right)$$

$$\alpha_1=I^2_{inj}\cos\left(\Delta\phi_2\right)-I^1_{inj}\cos\left(\Delta\phi_1\right)\qquad\beta_1=I^2_{inj}\sin\left(\Delta\phi_2\right)-I^1_{inj}\sin\left(\Delta\phi_1\right)$$

$$\alpha_2=I^4_{inj}\cos\left(\Delta\phi_4\right)-I^3_{inj}\cos\left(\Delta\phi_3\right)\qquad\beta_2=I^4_{inj}\sin\left(\Delta\phi_4\right)-I^3_{inj}\sin\left(\Delta\phi_3\right)$$

Medida de las derivadas

- Modificación del circuito



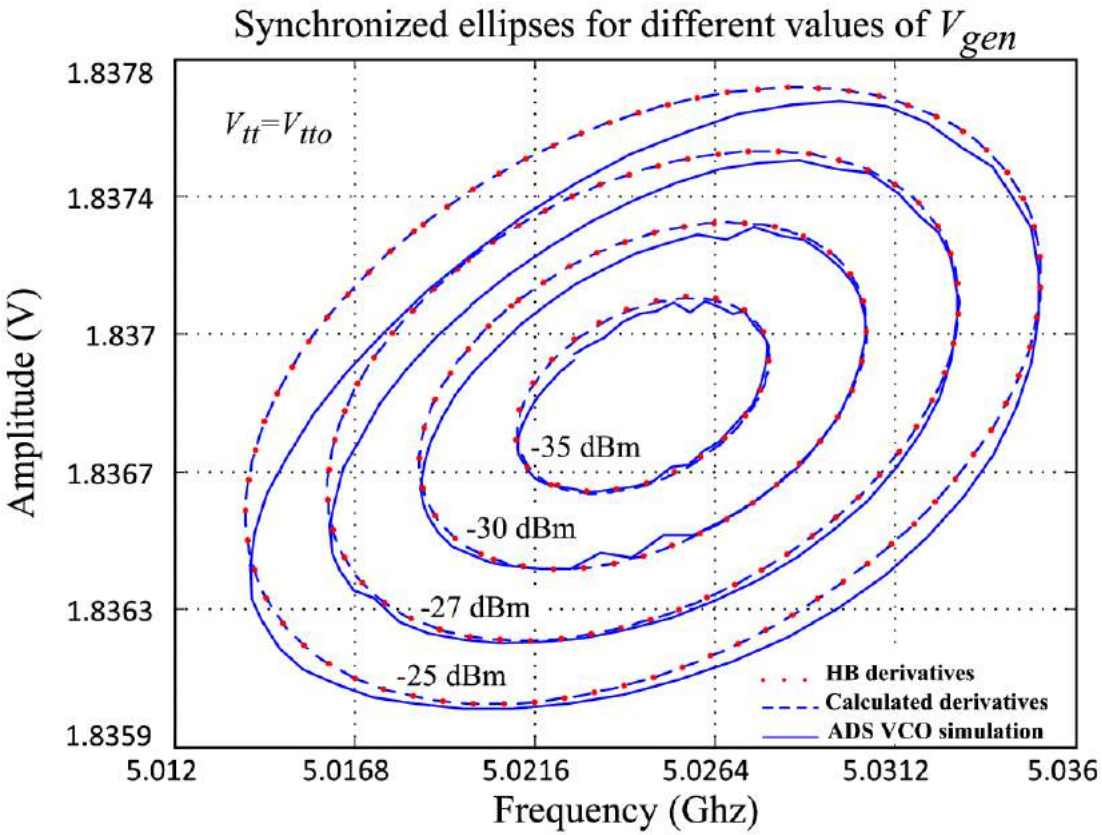
Medida de las derivadas

- Validación

Derivada	Valor de Referencia Sonda Medida	Valor Método Propuesto	Error de Magnitud (%)	Error de fase (%)	Error total (%)
$\frac{\delta Y_s}{\delta V}$	$2.328e-2\angle-19.26^o$	$2.281e-2\angle-17.93^o$	2.02	6.93	3.07
$\frac{\delta Y_s}{\delta \omega}$	$7.724e-12\angle 90.24^o$	$8.041e-12\angle 91.25^o$	4.12	1.12	4.50
$\frac{\delta Y_s}{\delta V_{tt}}$	$6.126e-4\angle-83.66^o$	$6.303e-4\angle-84.04^o$	2.88	0.46	2.96
$\frac{\delta Y_s}{\delta I^i_{inj}}$	$5.935e-1\angle-91.87^o$	$6.176e-1\angle-90.53^o$	4.06	1.46	4.71
$\frac{\delta Y_s}{\delta I^r_{inj}}$	$4.053e-1\angle 92.30^o$	$4.251e-1\angle 92.26^o$	4.86	0.04	4.86

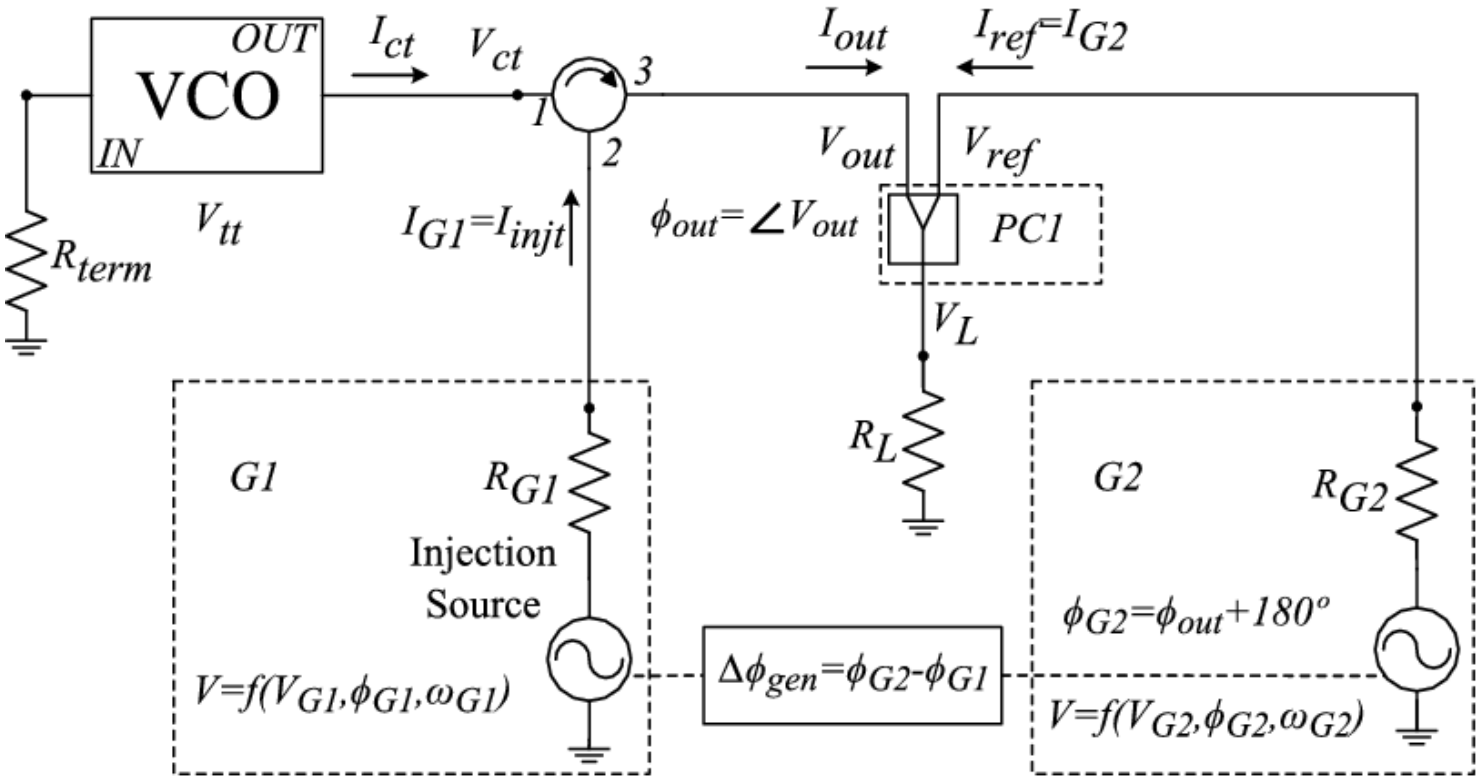
Medida de las derivadas

- Validación



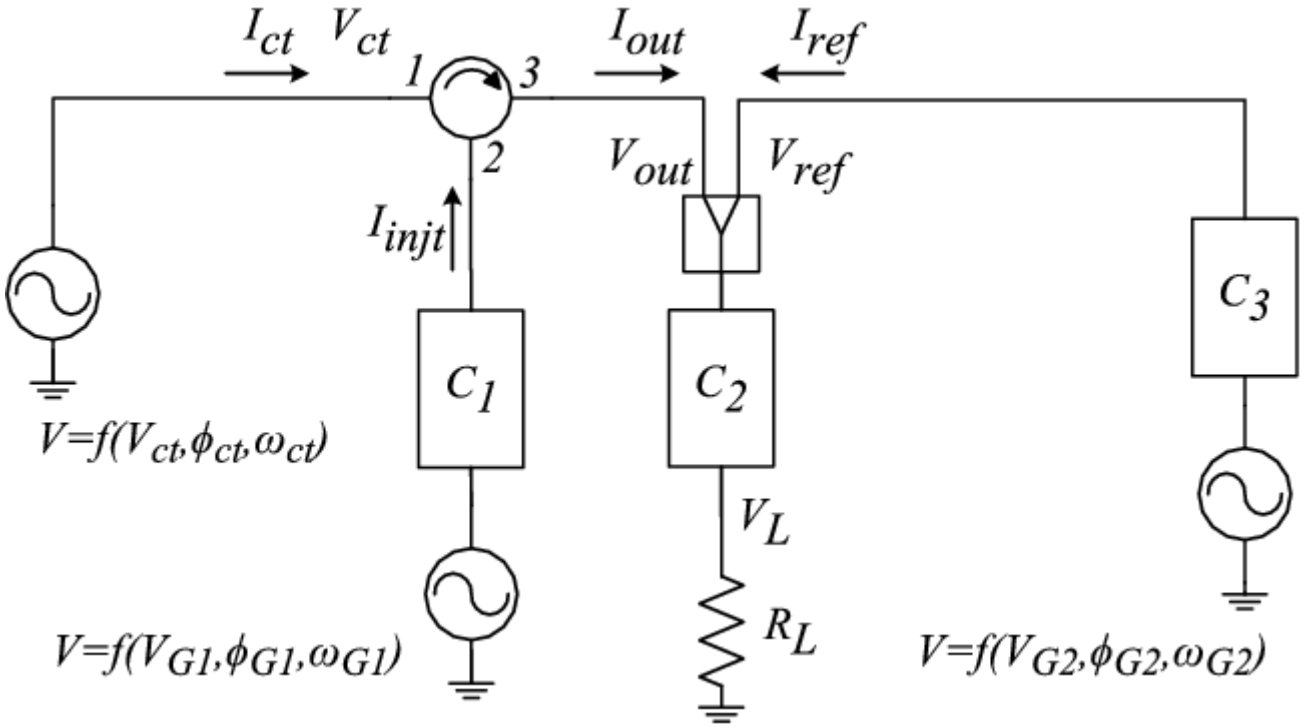
Medida de las derivadas

- Procedimiento de medida



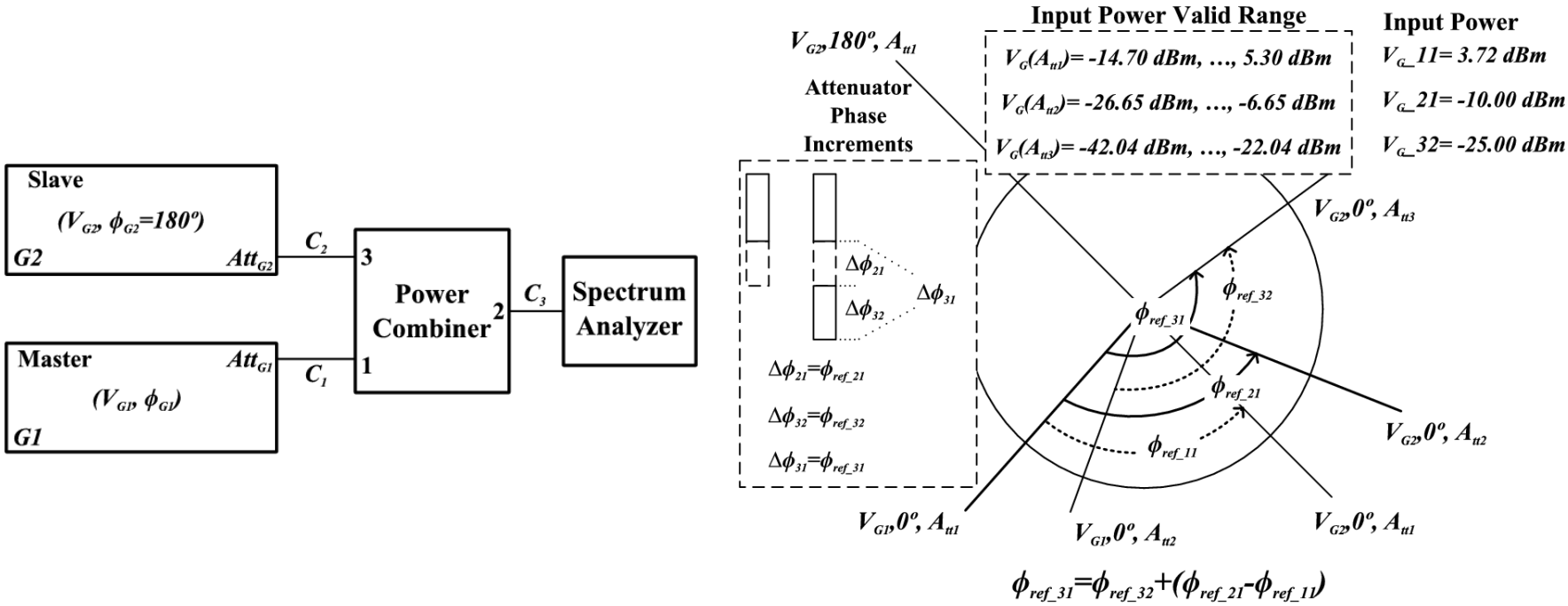
Medida de las derivadas

- Procedimiento de medida



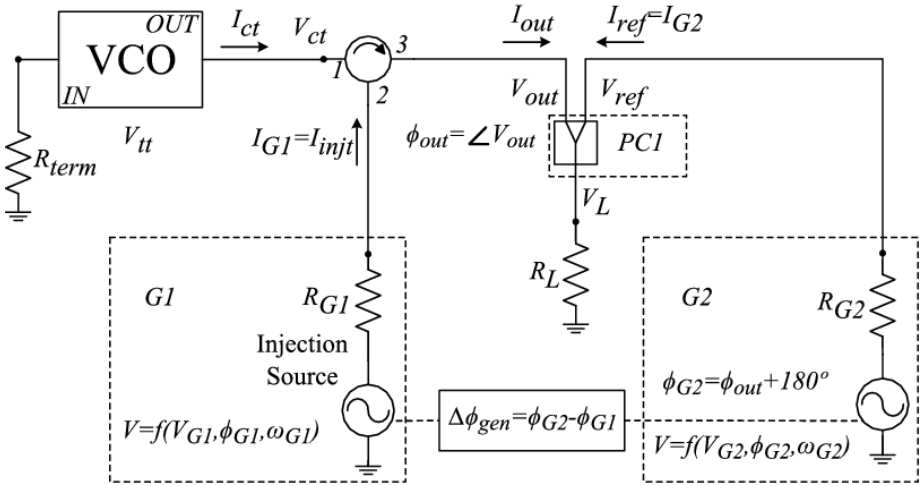
Medida de las derivadas

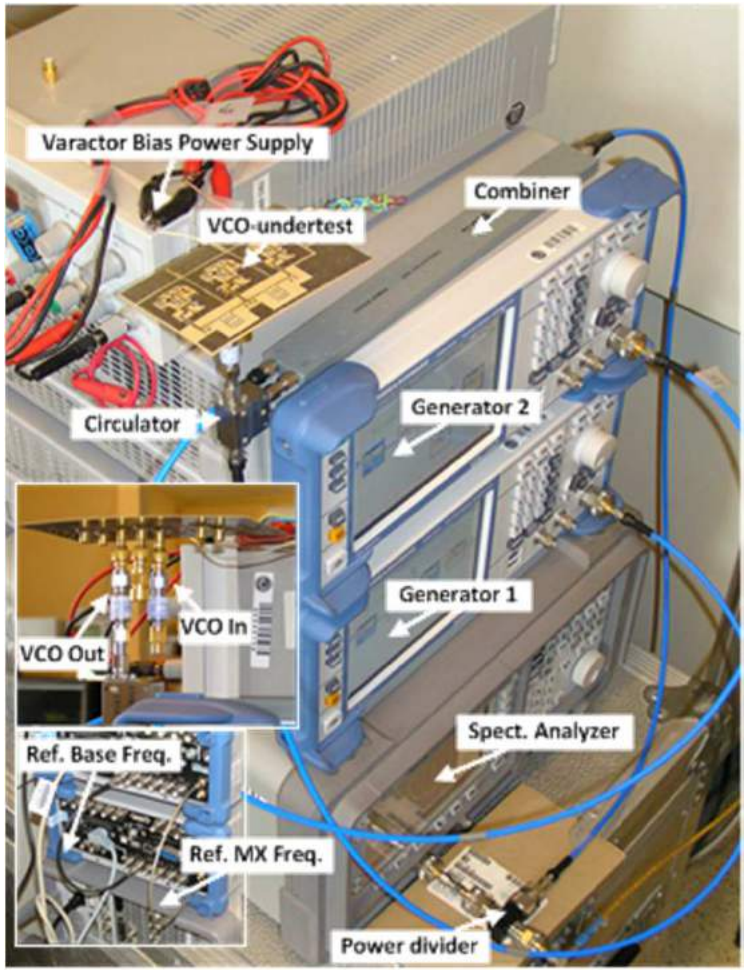
- Montaje final
  - Calibración de los generadores



Medida de las derivadas

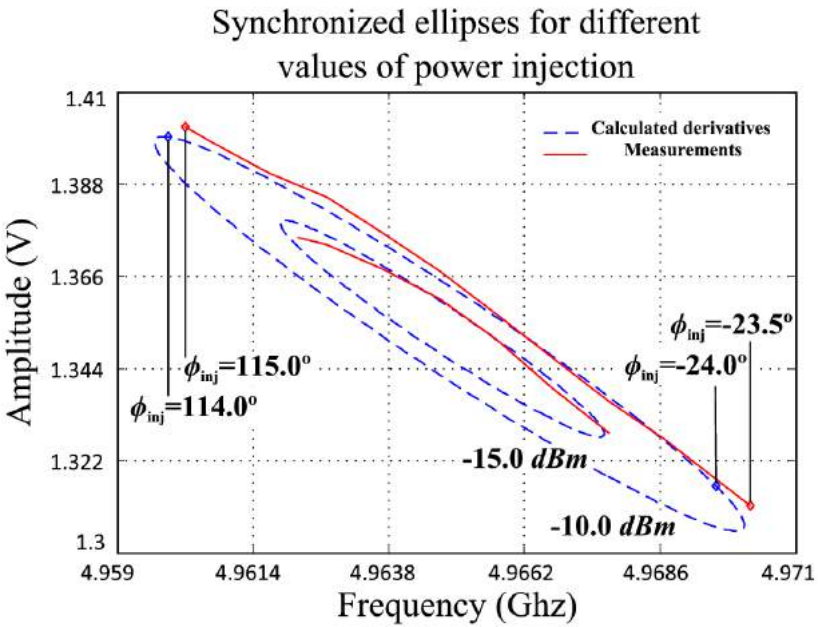
- Montaje final
  - Sistema de medida

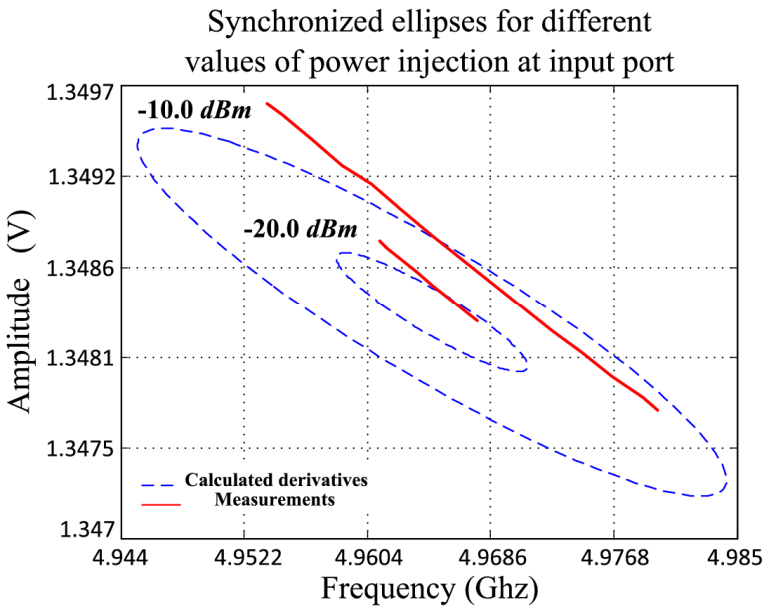




Medida de las derivadas

- Resultados





Aportaciones

- Aportaciones
  - Aplicación de la FSA a los arrays bidimensionales
  - Ajuste de la formulación
  - Construcción y medida de un array
  - Medida de derivadas de un VCO

Líneas futuras

- Líneas futuras
  - Extender y generalizar la aplicación de la FSA a nuevas configuraciones
  - Desarrollo de nuevas estrategias que permitan adaptar las expresiones para poder emplearlas en aquellos circuitos que no cumplan con los requerimientos de la FSA